Wakefields in plasmaguide

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Introduction

- Processes, which take place in plasmaguide when relativistic electron beam pass into it.

- We Assume that:
  - The bunch shape doesn’t change.
  - The plasma is isotropic, homogeneous and cold.

- Calculations done for gaussian, rectangular and parabolic bunches.

Electromagnetic Fields in Infinite Plasma

$j_z = Qv_z\delta(x)\delta(y)\delta(z - v_z t)$ Current density

The equations of Maxwell

$$rot\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}$$

$$rot\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$div\vec{B} = 0$$

$$div\vec{D} = 4\pi\rho$$

$$D_i(\omega,\vec{k}) = \varepsilon_{ij}E_j(\omega,\vec{k})$$

$$\varepsilon_{ij} = \delta_{ij}(1 - \frac{\omega_p^2}{\omega^2})$$

The modification of Fourier $(\vec{r}, t \rightarrow \vec{k}, \omega)$

$$E_z = -Qk_p{}^2 K_0(k_p r)\eta(\tau)\cos(\omega_p\tau)$$

where

$$k_p = \frac{\omega_p}{v_z},\quad \omega_p = \sqrt{\frac{4\pi ne}{m}},\quad r = \sqrt{x^2 + y^2}$$

$$\tau = t - \frac{z}{v_z},\qquad \eta(\tau) = 1 \rightarrow \tau > 0$$

$$\eta(\tau) = 0 \rightarrow \tau < 0$$

Electromagnetic Fields in Plasmaguide

$$\Delta E_z + \frac{\omega^2}{c^2}\varepsilon E_z = -\frac{4\pi i\omega}{c^2} j_z + \frac{4\pi}{i\omega}\frac{\partial}{\partial z} div\vec{j} \qquad E_z |_R = 0$$

$$E_z = -\frac{2Qk_p{}^2}{R^2}\eta(\tau) Cos\,\omega_p \tau \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r)}{J_1{}^2(\lambda_n R)(k_p{}^2 + \lambda_n{}^2)}$$

$$j_z = \frac{Qv_z}{2\pi r}\delta(r - r')\delta(z - v_z t) \quad E_z = -\frac{2Qk_p^2}{R^2}\eta(\tau) Cos\,\omega_p \tau \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r')J_0(\lambda_n r)}{J_1{}^2(\lambda_n R)(k_p^2 + \lambda_n^2)}$$

For any distribution

$$j_z = \frac{Q}{2\pi} P_1(r) P_2(\tau)$$

Wakefield

$$E_z' = 2\pi \int_0^{\infty} \int_{-\infty}^{\tau} r' E_z(r, r', \tau - \tau') P_1(r') P_2(\tau') d\tau' dr'$$

Results

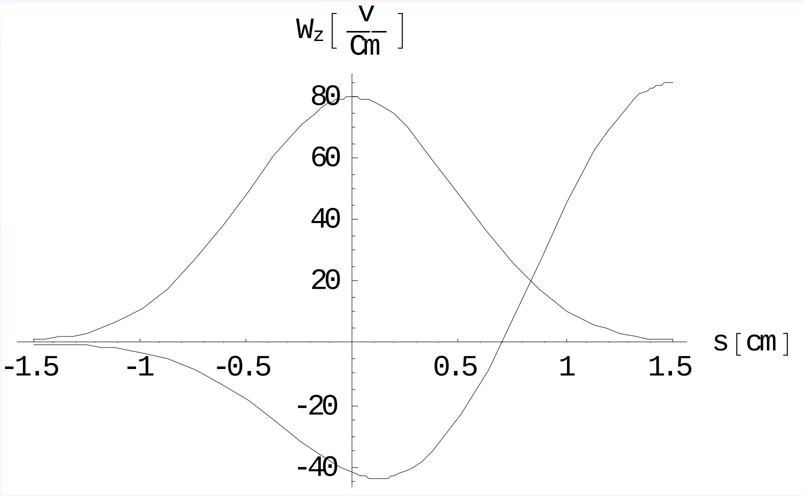
For Gaussian distribution $P_1(r) = \frac{1}{\sqrt{2\pi}\sigma_r} Exp(-\frac{r^2}{2\sigma_r^2})$ $P_2(s) = \frac{1}{\sqrt{2\pi}\sigma_s} Exp(-\frac{s^2}{2\sigma_s^2})$

$$w_z = -\frac{2k^2{}_p}{R^2\sigma^2} \int_0^\infty \int_{-\infty}^s r' Cos\frac{\omega_p}{c}(s-s') \sum_{n=1}^\infty \frac{J_0(\lambda_n r')J_0(\lambda_n r)}{J_1{}^2(\lambda_n R)(k_p{}^2+\lambda_n{}^2)} \cdot Exp(-\frac{r'^2}{2\sigma^2}) \cdot Exp(-\frac{s'^2}{2\sigma^2}) ds' dr'$$

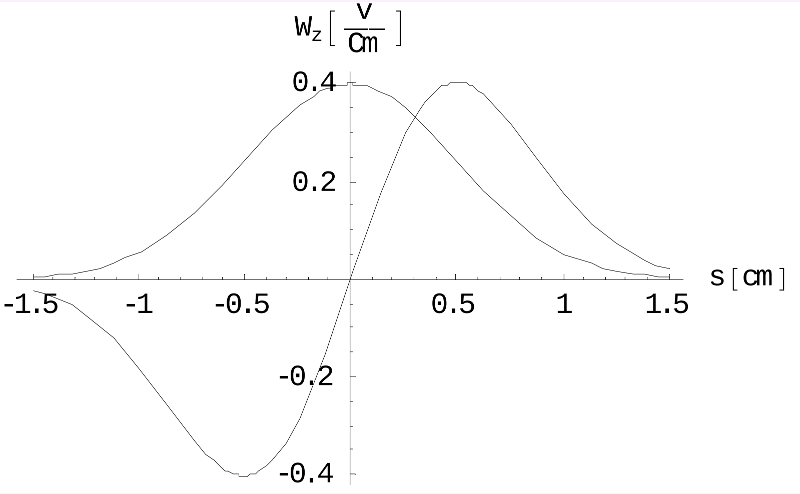
$R = 5cm, \quad \sigma = 0.5cm$

1. $n = 10^{12}\ cm^{-3} \rightarrow \omega_p = 10^{10}\ s^{-1}$

2. $n = 10^{16}\ cm^{-3} \rightarrow \omega_p = 10^{12}\ s^{-1}$



$\lambda \approx \sigma$



$\lambda \prec\prec \sigma$

Results

For rectangular distribution

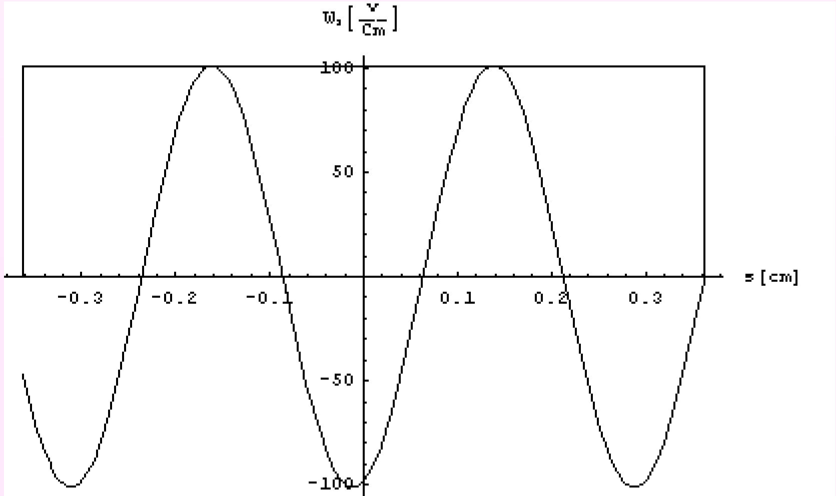
$$P(s) = \begin{cases} P_0(s) & -a \prec s \prec a \\ 0 & |s| \succ a \end{cases}$$

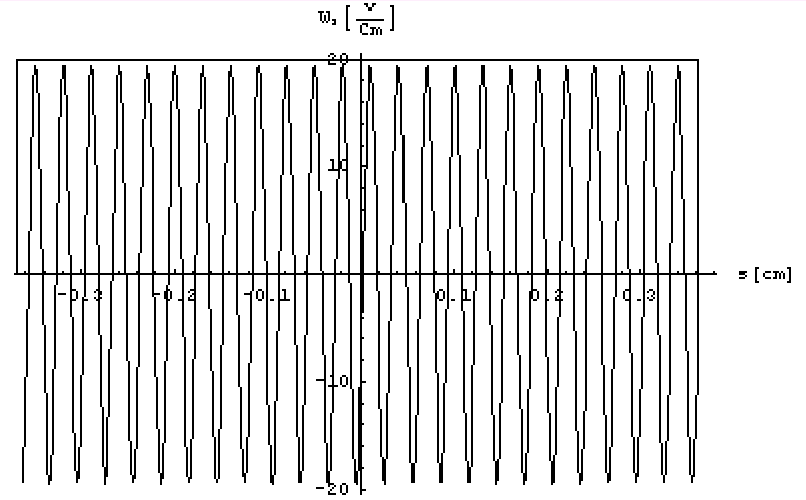
$$P_0(s) = \frac{1}{2a}, \qquad a = \sqrt{3}\sigma$$

$$w_z = -\frac{2k_p^2}{R^2} \int_0^{\infty} \int_{-\infty}^{s} r' Cos \frac{\omega_p}{c}(s-s') \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r') J_0(\lambda_n r)}{J_1^2(\lambda_n R)(k_p^2 + \lambda_n^2)} \cdot P_0 ds' dr'$$

1. $n = 10^{12}\ cm^{-3} \rightarrow \omega_p = 10^{10}\ s^{-1}$

2. $n = 10^{16}\ cm^{-3} \rightarrow \omega_p = 10^{12}\ s^{-1}$





Results

For parabolic distribution

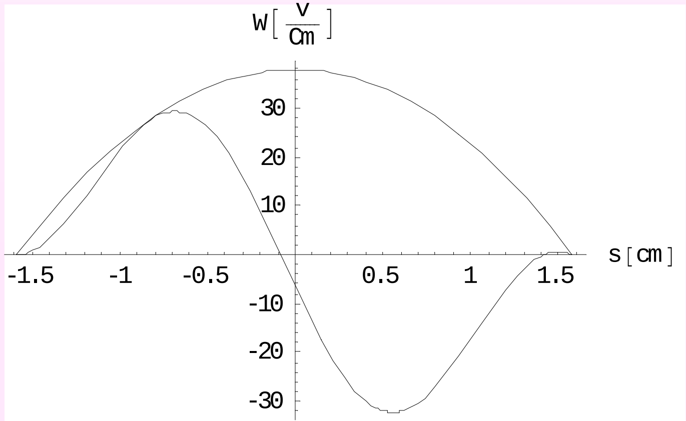
$$P(s)=\begin{cases}P_0(s) & -a \prec s \prec a\\ 0 & |s| \succ a\end{cases}$$

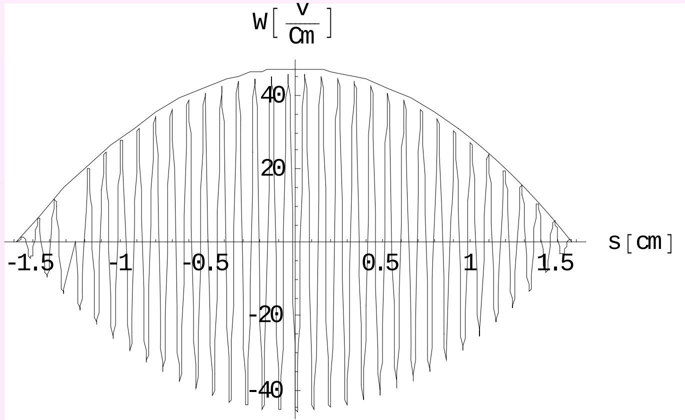
$$P_0(s) = -As^2 + B$$

$$w_z = -\frac{2k^2{}_p}{R^2}\int_0^{\infty}\int_{-\infty}^{s} r' Cos\frac{\omega_p}{c}(s-s')\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r')J_0(\lambda_n r)}{J_1{}^2(\lambda_n R)(k_p{}^2+\lambda_n{}^2)}\cdot P_0 ds' dr'$$

1. $n = 10^{12}\, cm^{-3} \rightarrow \omega_p = 10^{10}\, s^{-1}$

2. $n = 10^{16}\, cm^{-3} \rightarrow \omega_p = 10^{12}\, s^{-1}$





Results

$\sigma_s = 0.5cm,$

$\sigma_r = 0.1cm,$

$s = 0,$

$\omega = 10^{12}\,Hz,$

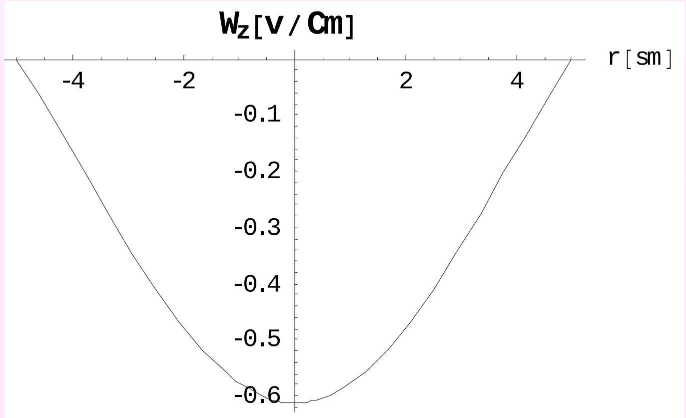
$n_p = 10^{12}\,cm^{-3}$

$$E'_z = 2\pi \int_0^{\infty} \int_{-\infty}^{\tau} r' E_z(r, r', s - s') P(s', r') ds' dr'$$

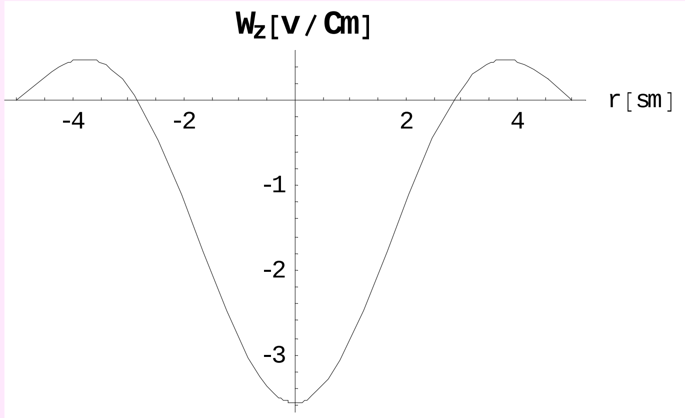
$$E_z = -\frac{2Qk_p^2}{R^2} \eta(\tau) Cos\omega_p \tau \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r') J_0(\lambda_n r)}{J_1^{\,2}(\lambda_n R)(k_p^{\,2} + \lambda_n^{\,2})}$$

$$P(s, r) = \frac{1}{2\pi \sigma_r \sigma_s} \left(Exp(-\frac{s^2}{2\sigma_s^{\,2}}) - Exp(-\frac{r^2}{2\sigma_r^{\,2}}) \right)$$

1 mod

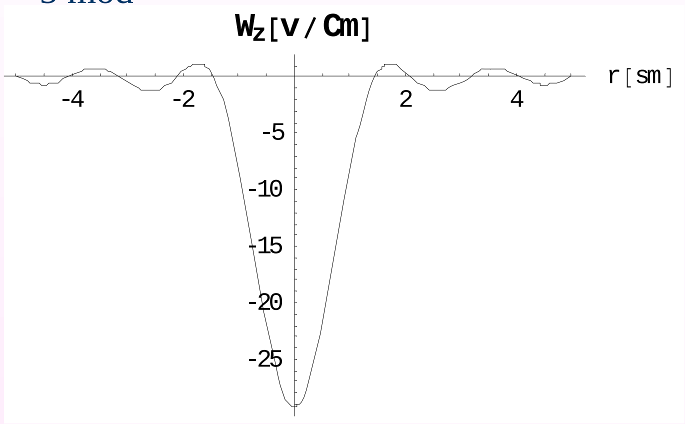


2 mod

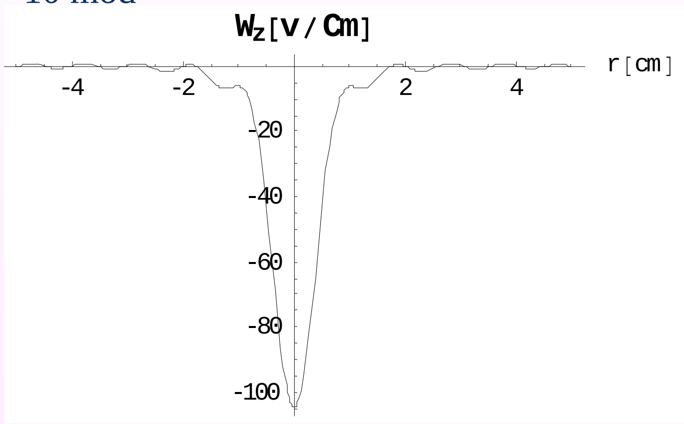


Results

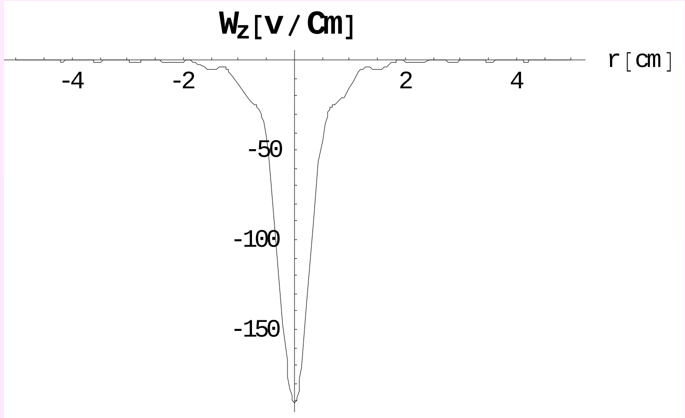
5 mod



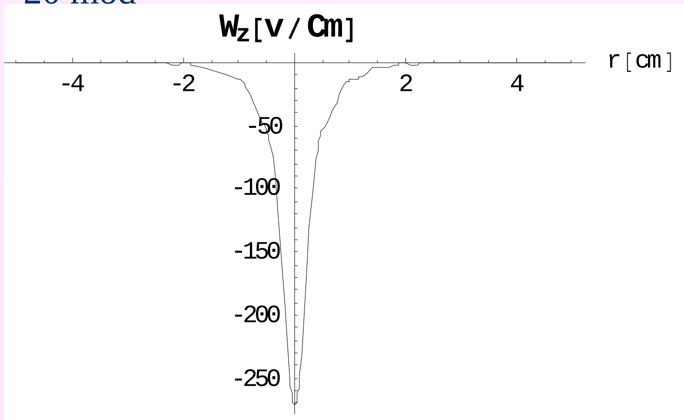
10 mod



15 mod

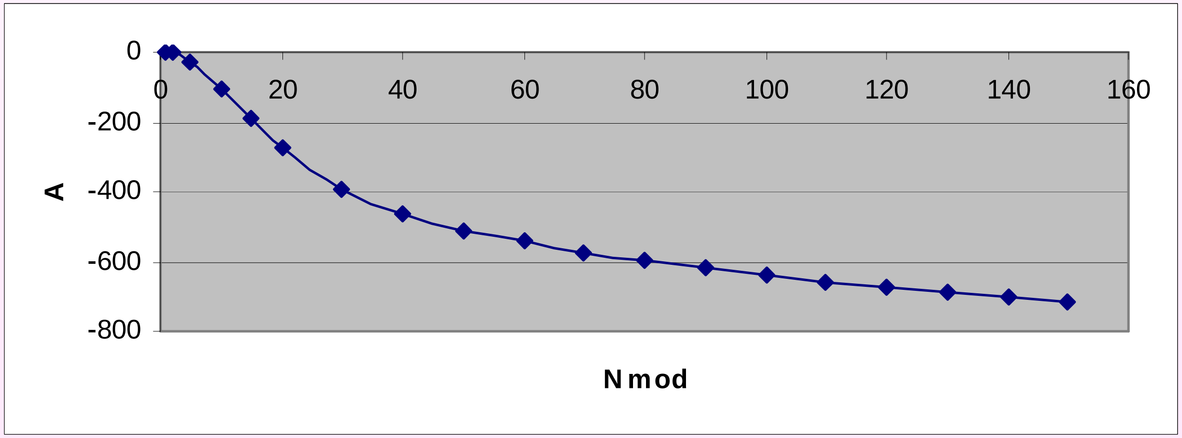


20 mod



Results

A	N mod
1	-0,6
2	-3,5
5	-29
10	-104,4
15	-190,6
20	-271
30	-391,1
40	-462,9
50	-508,9
60	-543,6
70	-572
80	-597,3
90	-619,4
100	-639,1
110	-656,9
120	-673,2
130	-688,2
140	-702
150	-715



Thank You

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