# Wakefields for Sinusoidal Trajectory in Undulator

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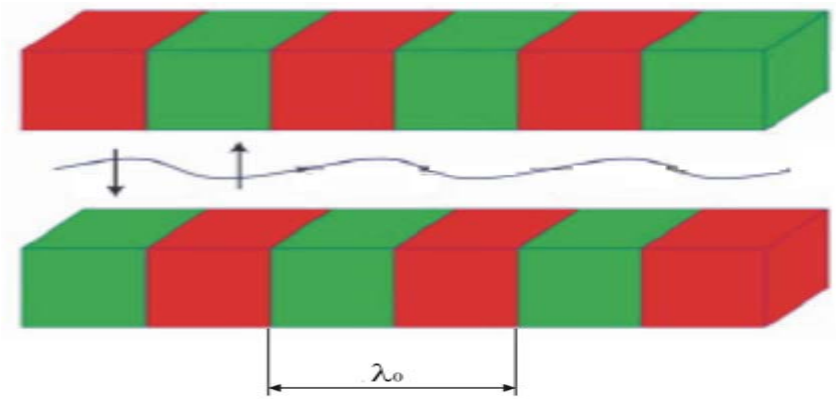
# Electron Trajectory in Undulator




Equation of trajectory ➡

$$x(t) = \frac{K}{\gamma}\frac{\lambda_0}{2\pi}\sin\left(\frac{2\pi\bar{\beta}ct}{\lambda_0}\right)$$

$$y(t) = 0$$

$$z(t) = \bar{\beta}ct - \frac{K^2\lambda_0}{16\pi\gamma^2}\sin\left(\frac{4\pi}{\lambda_0}\bar{\beta}ct\right)$$

# Culculation Method

Wave equation for fields component

$$\Delta E_{z\omega} + \frac{\omega^2}{c^2}\varepsilon E_{z\omega} = \frac{4\pi\omega}{c^2} j_{z\omega} + \frac{4\pi}{\varepsilon}\frac{\partial \rho_\omega}{\partial z}$$

$$\Delta H_{z\omega} + \frac{\omega^2}{c^2}\varepsilon H_{z\omega} = -\frac{4\pi}{c} rot_z \vec{j}_\omega$$

Searching the solution in this form →

$$E_{z\omega} = \sum_{n=0}^{\infty} E_{\omega n}(z)\Psi_n(x,y)$$

$$H_{z\omega} = \sum_{n=0}^{\infty} H_{\omega n}(z)\hat{\Psi}_n(x,y)$$

$\Psi_n(x,y), \hat{\Psi}_n(x,y)$ → Eigen functions

Boundary conditions →

$$\Psi_n(x,y)\big|_s = 0$$

$$\left.\frac{\partial \hat{\Psi}_n(x,y)}{\partial n_0}\right|_s = 0$$

# *Calculation Method*

$$E_{\omega n}(z) = -\frac{q\lambda_n^2}{\varepsilon\omega\gamma_n}\int_t v_z(t)\Psi_n(x(t),y(t))\,e^{-i\gamma_n|z-z(t)|-i\omega t}dt +$$

$$+\frac{q}{i\omega\varepsilon}\int_t \left(\vec{v}(t)\vec{\nabla}\Psi_n(x(t),y(t))\right)e^{-i\gamma_n|z-z(t)|-i\omega t}\,sign|z-z(t)|dt$$

$$H_{\omega n}(z) = -\frac{iq}{c\hat{\gamma}_n}\int_t \left[\vec{v}(t)\vec{\nabla}\Psi_n(x(t),y(t))\right]_z e^{-i\hat{\gamma}_n|z-z(t)|}dt$$

Leontovitch condition ➡ $\vec{E}_\tau = \zeta\cdot\left[\vec{H}_\tau\times\vec{n}\right]$