RELATION BETWEEN EIGENVALUES AND IMPEDANCE IN THE RESISTIVE ROUND PIPE

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26.02.2009

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Discoveries using **L**ight **E**mission

CONTENT

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2. Single-layer tube monopole resistive impedance. Leontovich boundary conditions.
3. Relation between impedance and eigenvalues.
4. Modal analysis of the monopole impedance
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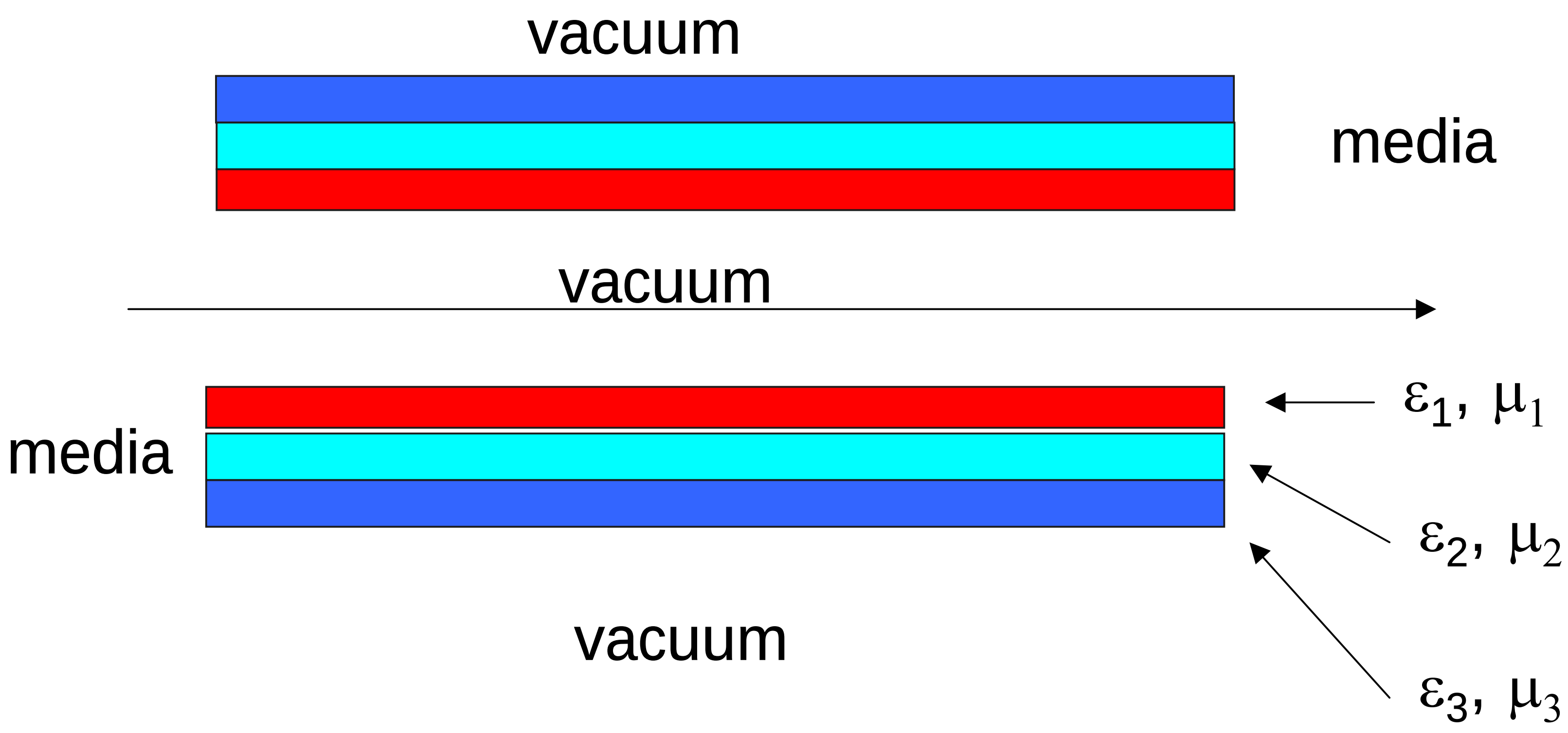
1. Determination of the multi-layer tube eigenfrequencies. Common equation.



Mode expressions.
Fundamental solutions of the homogeneous Maxwell equations

$$E_{mr}^{i} = \left(A_m^{(i)} \frac{jp}{\nu_i} Z'_m(\nu_i r) - B_m^{(i)} \frac{m\mu'_i k_0}{\nu_i^2 r} Z_m(\nu_i r) \right) F_m$$

$$E_\theta = -\left(A_m^{(i)} \frac{mp}{\nu_i^2 r} Z_m(\nu_i r) + B_m^{(i)} \frac{j\mu'_i k_0}{\nu_i} Z'_m(\nu_i r) \right) F_m$$

$$E_z = A_m^{(i)} Z_m(\nu_i r) F_m$$

$$Z_0 H_r = \left(A_m^{(i)} \frac{m k_i^2}{\mu_i' k_0 \nu_i^2 r} Z_m(\nu_i r) + B_m^{(i)} \frac{jp}{\nu} Z_m'(\nu_i r) \right) F_m$$

$$Z_0 H_\theta = \left(A_m^{(i)} \frac{j k_i^2}{\mu_i' k_0 \nu_i} Z_m'(\nu_i r) + B_m^{(i)} \frac{mp}{\nu_i^2 r} Z_m(\nu_i r) \right) F_m$$

$$Z_0 H_z = B_m Z_m(\nu r) F_m$$

$$\boxed{F_m = \exp\{ j(pz - \omega t) \}}$$

$$Z_m(\nu_i r) = \begin{cases} J_m(\nu_i r) \\ H_m(\nu_i r) \end{cases}$$

$$\nu_i^2 = k_i^2 - p^2, \quad k_i^2 = \omega^2 \varepsilon_i \mu_i = k^2 \varepsilon_i' \mu_i'$$

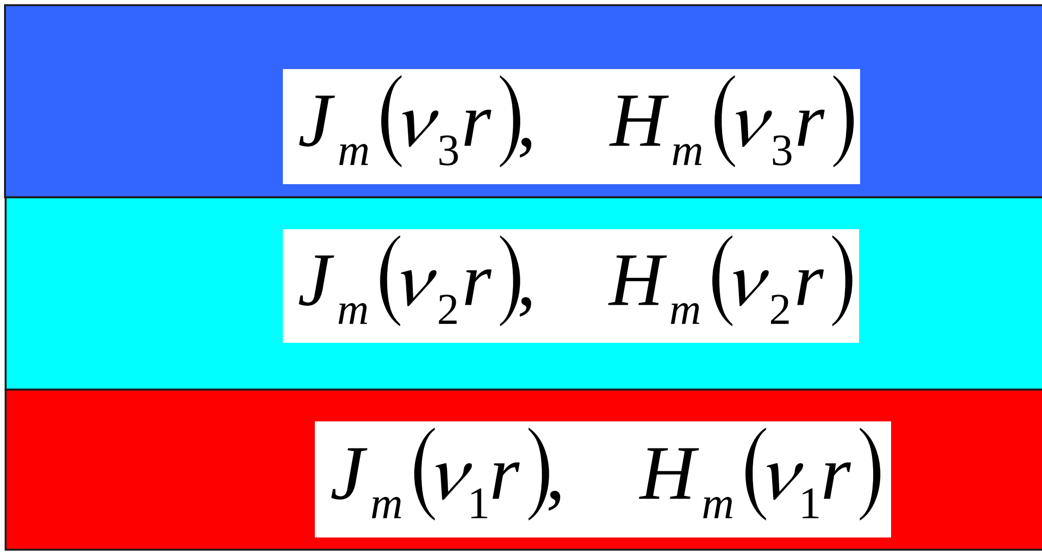
For vacuum

$$\varepsilon_i = \varepsilon_0, \quad \mu_i = \mu_0$$

For non-magnetic metal

$$\varepsilon_i = \varepsilon_0 - j\sigma_i / \omega, \quad \mu_i = \mu_0$$

vacuum $H_m(\nu_4 r)$ ← 2 coeff.



← 4 coeff.

← 4 coeff.

← 4 coeff.

vacuum $J_m(\nu_0 r)$ ← 2 coeff.

Dispersion equation

$$Det\left\{\hat{D}\right\} = 0$$

$$\hat{D} = \hat{Q}\hat{L}^{out}\left(\nu_0 a_{N+1}\right) - \hat{L}^{in}\left(\nu_0 a_1\right)$$

$$\hat{L}^{in}(\nu_0 r) = \begin{Bmatrix} -\dfrac{mp}{\nu_0^2 r} J_m(x) & 0 & -\dfrac{jk_0}{\nu_0} J'_m(x) & 0 \\ J_m(x) & 0 & 0 & 0 \\ -\dfrac{jk_0}{\nu_0} J'_m(x) & 0 & \dfrac{mp}{\nu_0^2 r} J_m(x) & 0 \\ 0 & 0 & J_m(x) & 0 \end{Bmatrix}$$

$$\hat{L}^{out}(\nu_0 r) = \begin{Bmatrix} 0 & -\dfrac{mp}{\nu_0^2 r} H_m(x) & 0 & -\dfrac{jk_0}{\nu_0} H'_m(x) \\ 0 & H_m(x) & 0 & 0 \\ 0 & -\dfrac{jk_0}{\nu_0} H'_m(x) & 0 & \dfrac{mp}{\nu_0^2 r} H_m(x) \\ 0 & 0 & 0 & H_m(x) \end{Bmatrix}$$

$$\hat{Q} = \overset{\frown}{Q}^{(1)} \overset{\frown}{Q}^{(2)} \cdots \overset{\frown}{Q}^{(N)}$$

$$Q^{(i)} = \frac{1}{d_i} \begin{Bmatrix} q_{11} & q_{12} & -\alpha q_{31} & \alpha q_{32} \\ 0 & q_{22} & \alpha q_{41} & -\alpha q_{42} \\ q_{31} & q_{32} & q_{11} & -q_{12} \\ q_{41} & q_{42} & 0 & q_{22} \end{Bmatrix}$$

$$d_i = H'_m(\nu_i a_{i+1}) J_m(\nu_i a_{i+1}) - H_m(\nu_i a_{i+1}) J'_m(\nu_i a_{i+1}) = j \frac{2}{\pi \nu_i a_{i+1}}$$

$$\alpha = \mu'_i / \varepsilon'_i$$

$$q_{11} = U_3^{(i)},\ q_{12} = \frac{mp}{a_i a_{i+1} \nu_i^2}\left(a_{i+1} U_2^{(i)} + a_i U_3^{(i)}\right),\ q_{22} = -U_2^{(i)},\ q_{31} = j\frac{mpU_1^{(i)}}{k\nu_i \mu' a_i}$$

$$q_{32} = j\frac{U_1^{(i)} m^2 p^2 + U_4^{(i)} a_i a_{i+1} k_c^2 \nu_i^2}{a_i a_{i+1} k \mu_i' \nu_i^3},\ q_{41} = j\frac{U_1^{(i)} \nu_i}{k\mu_i'},\ q_{42} = j\frac{U_1^{(i)} m_i p}{a_{i+1} k \mu' \nu_i}$$

$$U_1^{(i)} = H_m(\nu_i a_i) J_m(\nu_i a_{i+1}) - J_m(\nu_i a_i) H_m(\nu_i a_{i+1}),$$
$$U_2^{(i)} = H_m(\nu_i a_i) J_m'(\nu_i a_{i+1}) - J_m(\nu_i a_i) H_m'(\nu_i a_{i+1}),$$
$$U_3^{(i)} = H_m'(\nu_i a_i) J_m(\nu_i a_{i+1}) - J_m'(\nu_i a_i) H_m(\nu_i a_{i+1}),$$
$$U_4^{(i)} = H_m'(\nu_i a_i) J_m'(\nu_i a_{i+1}) - J_m'(\nu_i a_i) H_m'(\nu_i a_{i+1}).$$

For m=0

$$\hat{Q}^{(i)} = \frac{1}{d_i}\begin{Bmatrix} q_{11} & 0 & 0 & \alpha q_{32} \\ 0 & q_{22} & \alpha q_{41} & 0 \\ 0 & q_{32} & q_{11} & 0 \\ q_{41} & 0 & 0 & q_{22} \end{Bmatrix}$$

E - type

$$\hat{e}^{(i)} = \frac{1}{d_i}\begin{Bmatrix} q_{22} & \alpha q_{41} \\ q_{32} & q_{11} \end{Bmatrix}$$

H - type

$$\hat{h}^{(i)} = \frac{1}{d_i}\begin{Bmatrix} q_{11} & \alpha q_{32} \\ q_{41} & q_{22} \end{Bmatrix}$$

Single-layer tube monopole resistive impedance. Leontovich boundary conditions.

ε_0, μ_0

$$\vec{E}_t = \varsigma \cdot \vec{H}_t \times \vec{n}$$

$$\varsigma = \sqrt{\frac{\mu_1}{\varepsilon_1}} = (1 + j)\sqrt{\frac{\mu_1 \omega}{2\sigma_1}}$$

$$\omega << \sigma / \varepsilon_0$$

$$E_r = \left\{ \frac{jkF_z r}{2} + \frac{q}{2\pi r \varepsilon_0 c} \right\} \exp(-jkz)$$

$$E_z = F_z \exp(-jkz)$$

$$H_\theta = \left\{ \frac{j\omega \varepsilon_0 F_z r}{2} + \frac{q}{2\pi r} \right\} \exp(-jkz)$$

$$F_z = -\frac{Z_0 s_0}{2\pi a^2} F(\kappa)$$

$$F(\kappa) = \left(\frac{1-j}{\sqrt{\kappa}} + j\frac{\kappa}{2} \right)^{-1}$$

$$s_0 = \left(\frac{2ca^2\varepsilon_0}{\sigma} \right) \qquad \kappa = ks_0$$

Leontovich boundary conditions for the modes:

E - type

$$\frac{\tilde{\nu}_0 J_0(\tilde{\nu}_0)}{J_1(\tilde{\nu}_0)} = \frac{1-j}{2}\kappa^{3/2}$$

H - type

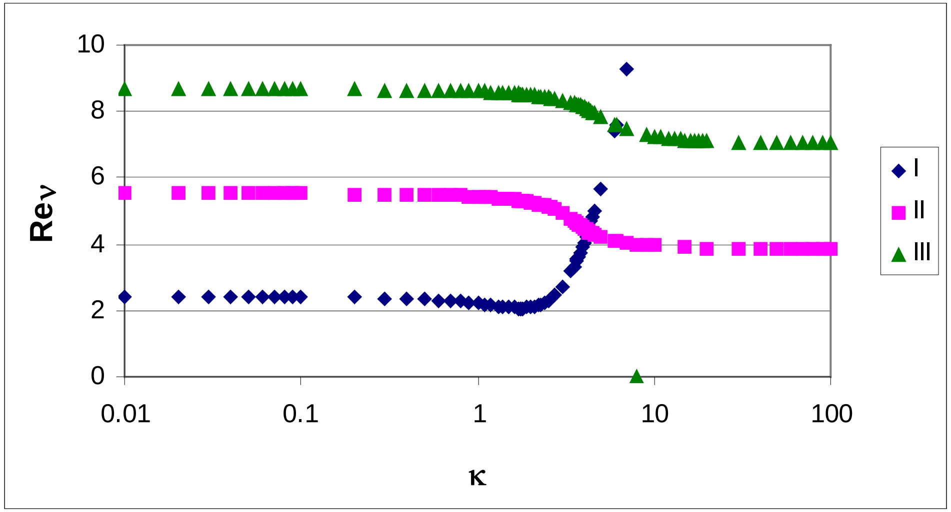
$$J_1(\hat{\nu}_0) = -\frac{1-j}{2\sqrt{\tilde{\kappa}}}\hat{\nu}_0 J_0(\hat{\nu}_0)$$

Relation to impedance

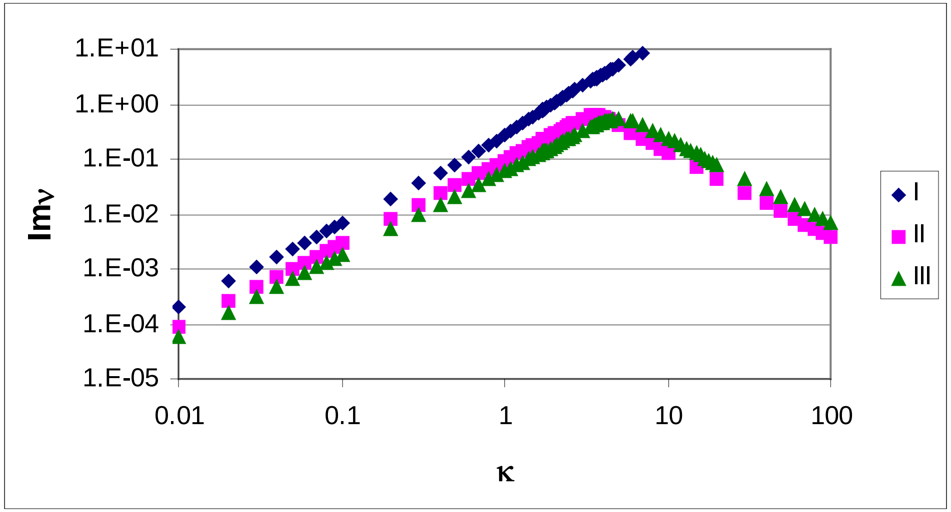
$$\frac{2j}{\kappa}\frac{J_0(\tilde{\nu}_0)}{J_2(\tilde{\nu}_0)} = \frac{1}{\frac{1-j}{\sqrt{|\kappa|}} + j\frac{\kappa}{2}} = F(\kappa)$$

$$\tilde{\kappa} = \kappa\frac{a^4}{s_0^4}$$

E - type

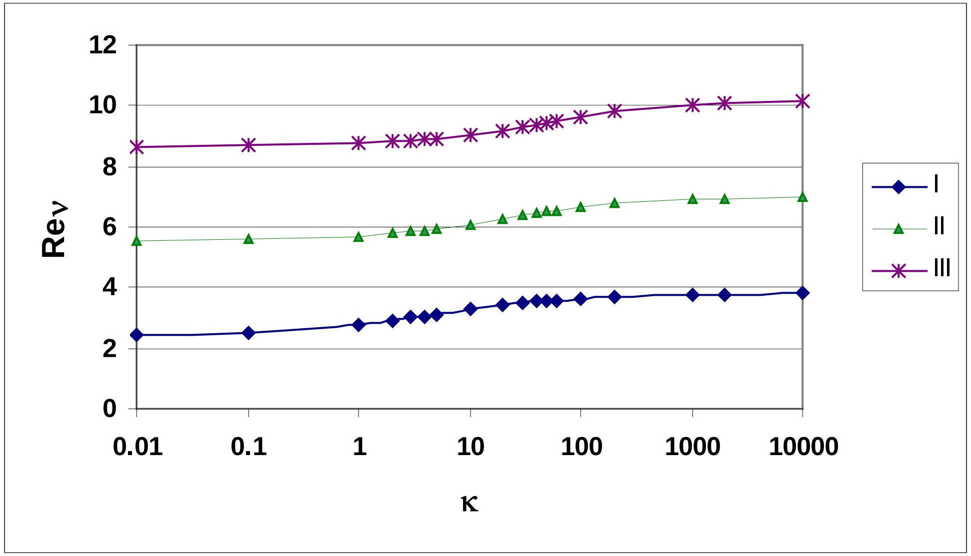


10
8
6
4
2
0
Reν
0.01
0.1
1
10
100
κ
I
II
III



1.E+01
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
Imν
0.01
0.1
1
10
100
κ
I
II
III

H - type



12
10
8
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0
Reν
0.01
0.1
1
10
100
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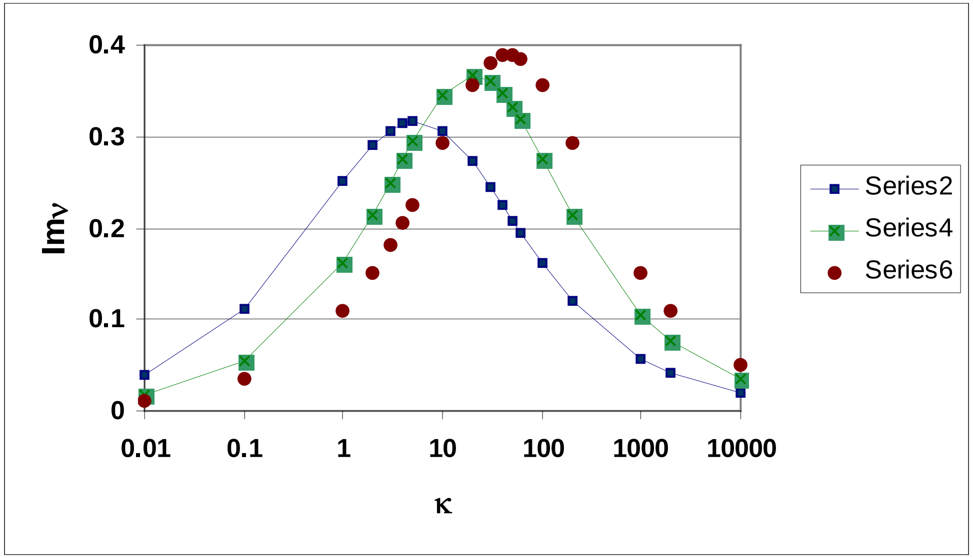
10000
κ
I
II
III



0.4
0.3
0.2
0.1
0
Imν
0.01
0.1
1
10
100
1000
10000
κ
Series2
Series4
Series6

PROPERTIES of ORTHOGONALITY

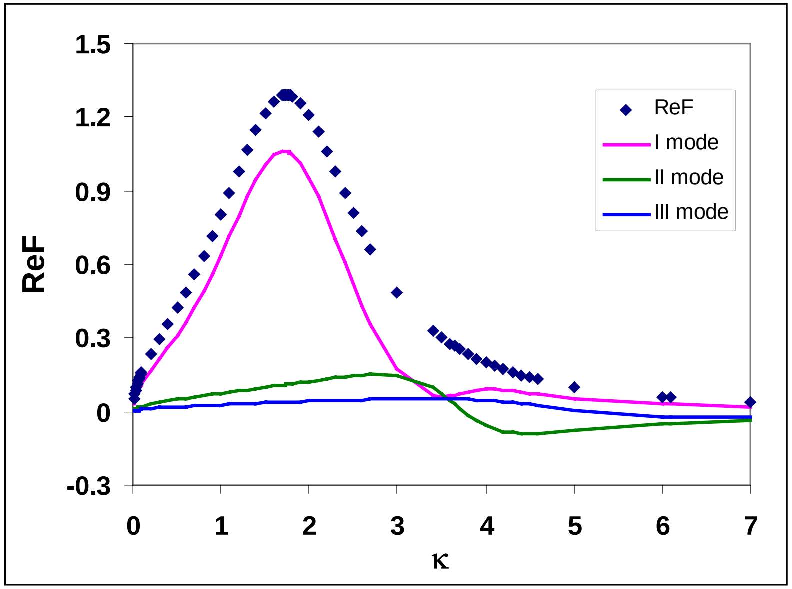
$$\int_0^a J_1\left(\nu_i \frac{r}{a}\right) J_1\left(\nu_j \frac{r}{a}\right) r dr = \begin{cases} \frac{a^2}{2}\left(J_1^2(\nu_i) - J_0(\nu_i) J_2(\nu_i)\right) & for\ i = j \\ 0 & for\ i \neq j \end{cases}$$

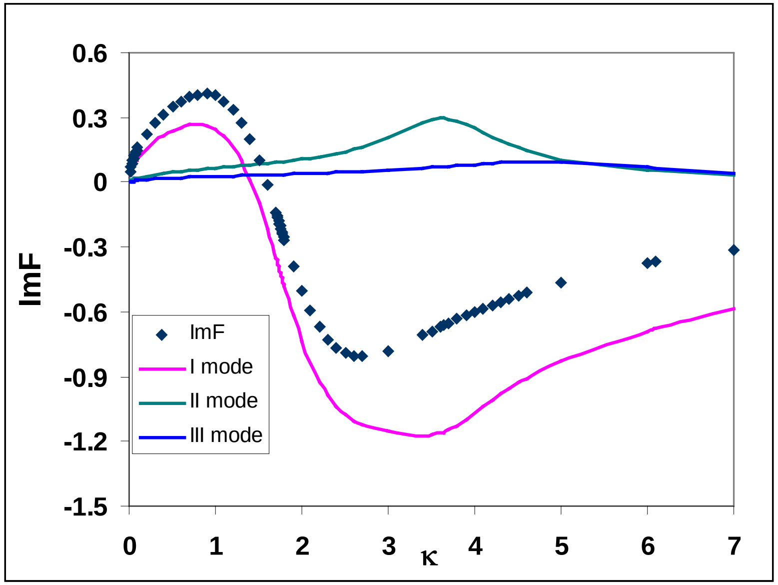
Series expansion

$$F(\kappa) = 2\frac{a}{r}\sum_{i=1}^{\infty} A_0^{(i)} \frac{J_1(\tilde{\nu}_i r/a)}{\tilde{\nu}_i}$$

$$A_0^{(i)} = \frac{1}{J_0(\tilde{\nu}_i)} \frac{1}{\frac{\tilde{\nu}_i^2}{\kappa^2} - \frac{1}{F(\kappa)}}$$

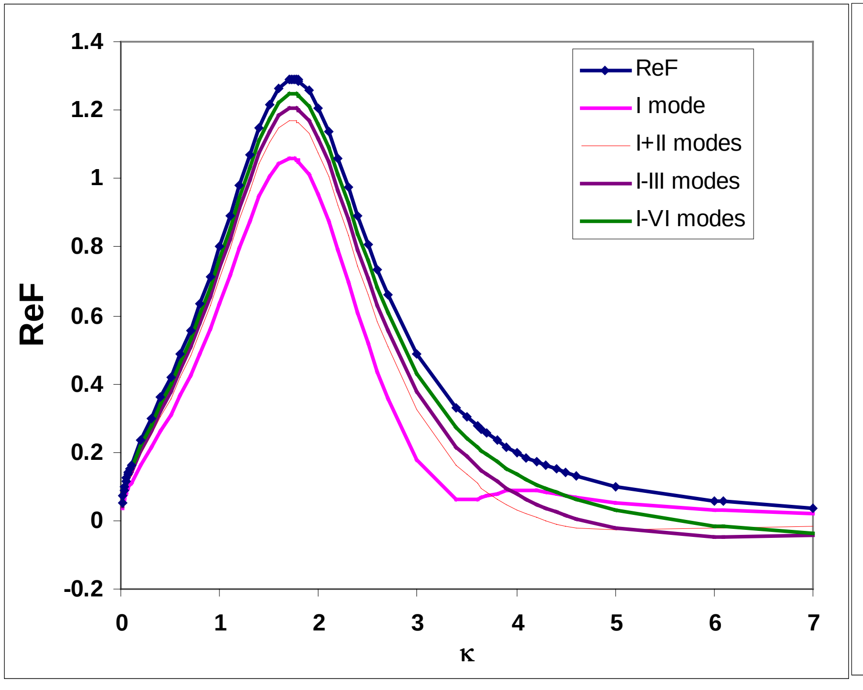
MODAL ANALYSIS

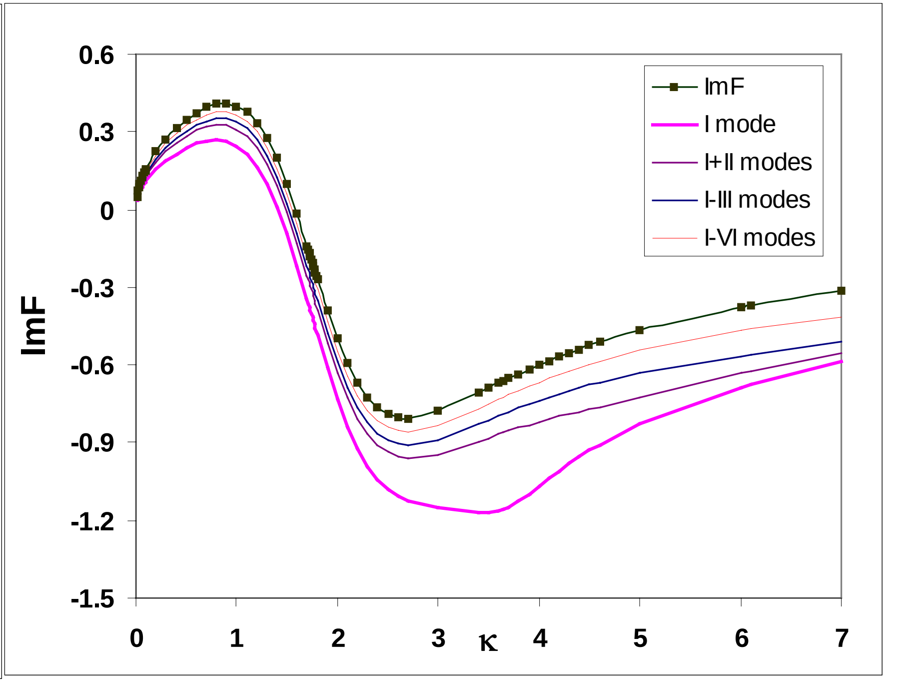




Distribution of the real and imaginary parts of the modal functions

Series convergence





$$F(\kappa) = 2\frac{a}{r}\sum_{i=1}^{\infty} A_0^{(i)} \frac{J_1\left(\tilde{\nu}_i\, r/a\right)}{\tilde{\nu}_i}$$

r=a

CONCLUSION

1. **The common method of the eigenvalues determination for the multi-layer round tube is obtained**
2. **For the single layer tube:**

The relation between impedance and eigenvalues is established.

The behaviors of the eigenvalues distribution versus frequency are determinated

An anomalous distribution of the fundamental E-mode is obtained. The resonance behaviors of impedance are conditioned by the fundamental mode peculiar properties

The modal series convergence is shown.