Transverse decoherence in the SLS storage ring

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Introduction

When a beam is kicked transversely from the closed orbit, it begins making betatron oscillations about the closed orbit. The oscillations can be observed with beam position monitors (BPM), which give the centroid of the beam. If the particles all have the same betatron tune the centroid motion is pseudo-harmonic. However, if the beam contains a spread of tunes, the motion will decohere as the individual betatron phases of the particles disperse.

Here we will consider decoherence due to two sources of betatron tune spread : The beam bunch may have an intrinsic betatron tune spread due to transverse nonlinearity, and there may be an additio-nal tune spread due to the energy spread of the beam which is coupled to betatron tune through the chromaticity.

Introduction

For solving these problems exactly we make some assumptions. In the case of transverse nonlinearity, we assume that the transverse distribution is Gaussian. This implicitly assumes that the distortion of phase space trajectories due to the nonlinearity is small. Also assume that the tune shift with betatron amplitude is a quadratic function.

For the case of decoherence due to chromaticity, we assume that the synchrotron motion is linear and the energy distribution is Gaussian. Also assume that the energy distribution is uncorrelated with the transverse distribution. Under this ussumption the decoherence due to chromaticity is completely independent of the transverse distribution. Noth that in this case we take into account the first and the second order chromaticitis.

Introduction

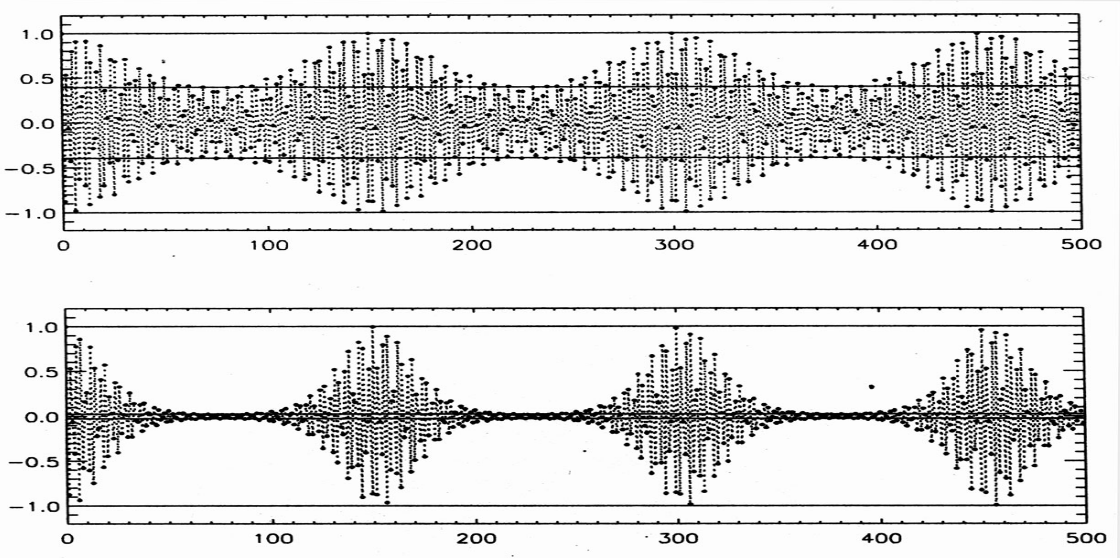
Note that in this direction some theoretical results were obtained

for example in

1. *R.E. Meller, A. W. Chao and others, "Decoherence of Kickid Beams".*
2. *A. Streun, "Measurement of electron beam rms energy spread from betatron oscillation decoherence/recoherence signals".*

Introduction

$$\nu = \nu_0 + \xi \frac{\Delta E}{E}. \quad < x(N) >= z \cdot e^{-\left(\frac{\xi \sigma_e}{\nu_s}\right)^2 (1-\cos 2\pi N \nu_s)} \cos(2\pi \nu_0 N + \Phi_0)$$

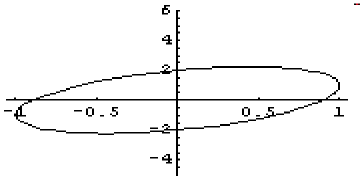


$$\sigma_e = \frac{\nu_s}{\xi}\sqrt{-\frac{1}{2}\ln R}, \quad R = \frac{X_{\min}}{X_{\max}}.$$

Theoretical model

Before the kick we have the following distribution in the betatron phase space:

$$\rho_\beta(x,x') = \frac{1}{2\pi\varepsilon} e^{-\frac{1}{2}\left(\frac{\gamma x^2+2\alpha x x'+\beta x'^2}{\varepsilon}\right)},$$



Putting $\bar{x} = \frac{x}{\sigma_x} = r\cos\varphi, \quad \bar{p} = \frac{\alpha x + \beta x'}{\sigma_x} = -r\sin\varphi, \quad \sigma_x = \sqrt{\beta\varepsilon}.$

we have

$$\rho(x,x')dxdx' = \rho(x(r,\varphi),x'(r,\varphi))\left|J(x_{r,\varphi},x'_{r,\varphi})\right|drd\varphi = \bar{\rho}(r,\varphi)drd\varphi.$$

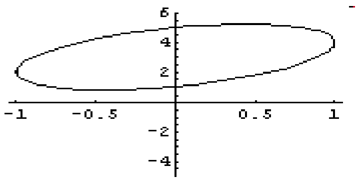
After the change of variables we obtain

$$\bar{\rho}_\beta(r,\varphi) = \frac{1}{2\pi} r e^{-\frac{1}{2}r^2}.$$

Theoretical model

After the kick we have

$$\rho(x, x' - \Delta x') = \frac{1}{2\pi\varepsilon} e^{-\frac{1}{2}\left(\frac{\gamma x^2 + 2\alpha x x' + \beta (x' - \Delta x')^2}{\varepsilon}\right)},$$



The distribution for the kicked beam is

$$\overline{\overline{\rho_{\beta}}}(r, \varphi) = \frac{1}{2\pi} r e^{-\frac{1}{2}\left(r^2 + z^2 + 2 r z \sin\varphi\right)}.$$

where $z = \dfrac{\beta \Delta x'}{\sigma_x}.$

The distribution in the synchrotron phase space has the following form and is invariant in time:

$$\rho_{\varepsilon}(\delta, \tau) = \frac{1}{2\pi} e^{-\frac{1}{2}\left(\delta^2 + \tau^2\right)}, \quad \delta = \frac{\Delta E / E}{\sigma_e}, \quad \tau = \frac{\Delta s}{\sigma_s}.$$

Theoretical model

According to our considerations we have the following expression for the betatron tune:

$$\nu = \nu_0 - \mu r^2 + \xi_1 \frac{\Delta E}{E} + \xi_2 \left(\frac{\Delta E}{E} \right)^2 .$$

Particles in longitudional phase spase propagate in time by

$$\begin{pmatrix} \delta(N) \\ \tau(N) \end{pmatrix} = \begin{pmatrix} \cos 2\pi\upsilon_s N & \sin 2\pi\upsilon_s N \\ \sin 2\pi\upsilon_s N & \cos 2\pi\upsilon_s N \end{pmatrix} \begin{pmatrix} \delta_0 \\ \tau_0 \end{pmatrix} .$$

And for the phase advanse we obtain

$$\varphi(N) - \varphi_0 = 2\pi \int_0^N \nu(n) dn = 2\pi\nu_0 N - 2\pi\mu N r^2 + A\delta_0^2 + B\tau_0^2 + C\delta_0\tau_0 + 2a\delta_0 + 2b\tau_0 .$$

Theoretical model

where

$$T = 2\pi\nu_s \mathrm{N}, \quad s = \sin T, \quad c = \cos T, \quad K_1 = \frac{\sigma_e \xi_1}{\nu_s}, \quad K_2 = \frac{\sigma_e^2 \xi_2}{\nu_s}$$

$$a = \frac{K_1 s}{2}, \quad b = \frac{K_1(1-c)}{2}, \quad A = \frac{K_2(T+sc)}{2}, \quad B = \frac{K_2(T-sc)}{2}, \quad C = \frac{K_2 s^2}{2}.$$

Theoretical model

For the distribution in 4-dimentional phase space we have

$$\rho(r,\varphi,\delta,\tau,N)=\overline{\overline{\rho}}_{\beta}(r,\varphi-2\pi\upsilon_0 N+2\pi\mu N r^2-(A\delta^2+B\tau^2+2a\delta+2b\tau+C\delta\tau))\cdot\rho_e(\delta,\tau).$$

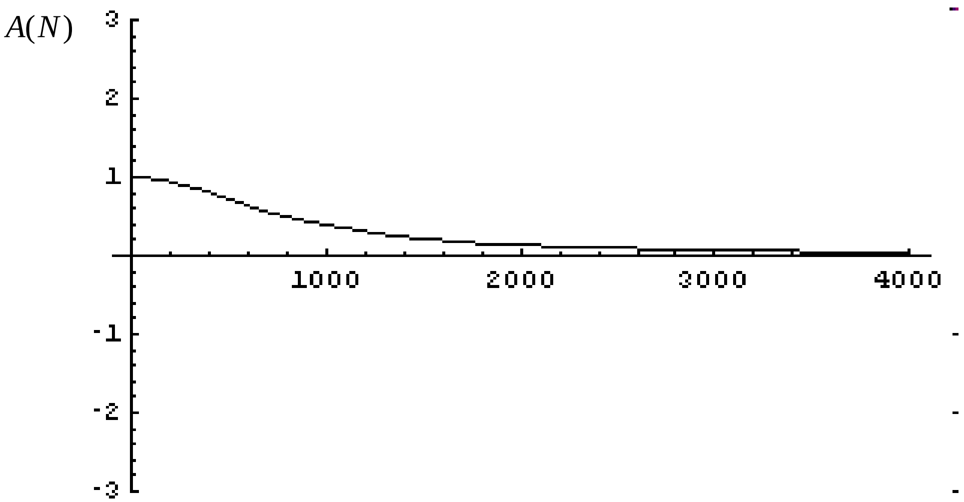
So for centroid displacement we get

$$<x(N)>=\frac{\sigma_x}{4\pi^2}\int_0^{\infty}\int_0^{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r\cos\varphi\cdot\rho(r,\varphi,\delta,\tau,N)\,dr\,d\varphi\,d\delta\,d\tau=$$

$$=z\sigma_x A(N)H(N)e^{-M(N)}\cos\left(P(N)+\Phi(N)+2\pi\upsilon_0 N-\frac{\pi}{2}\right).$$

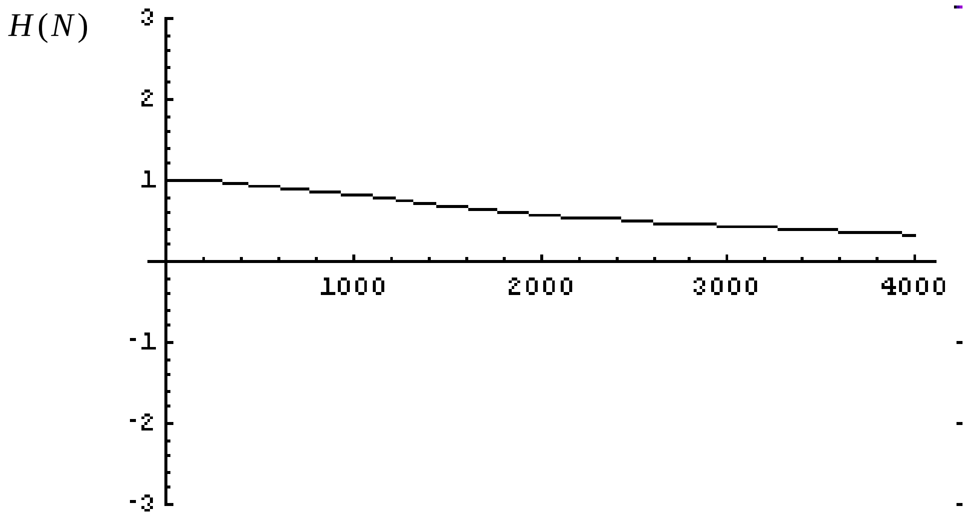
Theoretical model

$$A(\mathrm{N}) = \frac{1}{1+(4\pi\mu \mathrm{N})^2} e^{-\frac{z^2}{2}\cdot\frac{(4\pi\mu\mathrm{N})^2}{1+(4\pi\mu\mathrm{N})^2}}$$



Theoretical model

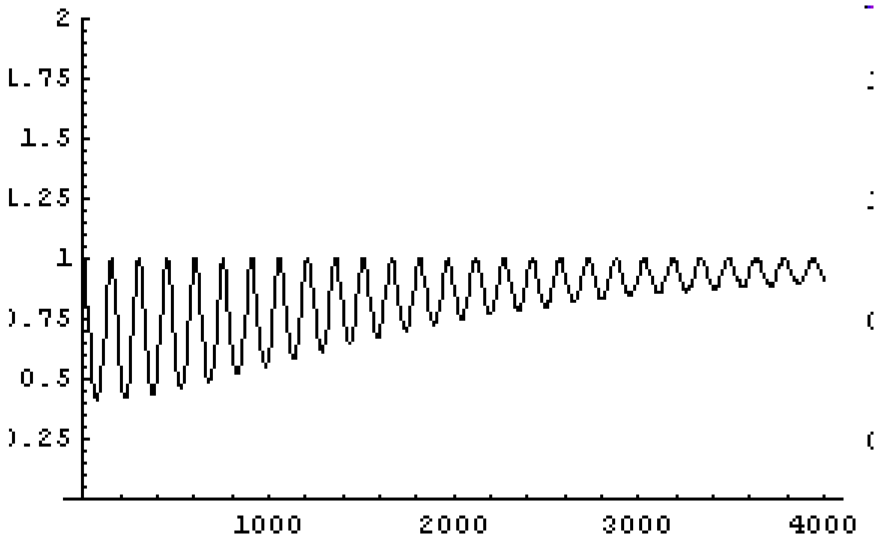
$$H(N) = \left[1 + 2K_2^2\left(T^2 + s^2\right) + K_2^4\left(T^2 - s^2\right)^2\right]^{-\frac{1}{4}}$$



Theoretical model

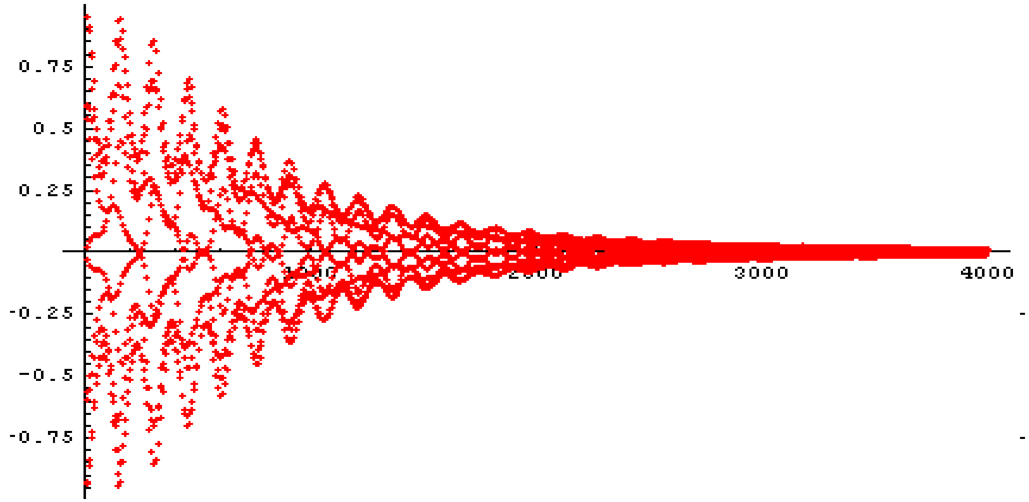
$$\mathrm{M}(\mathrm{N}) = \frac{K_1^2(1-c)}{1+K_2^2(T+s)^2}$$

$e^{-M(N)}$

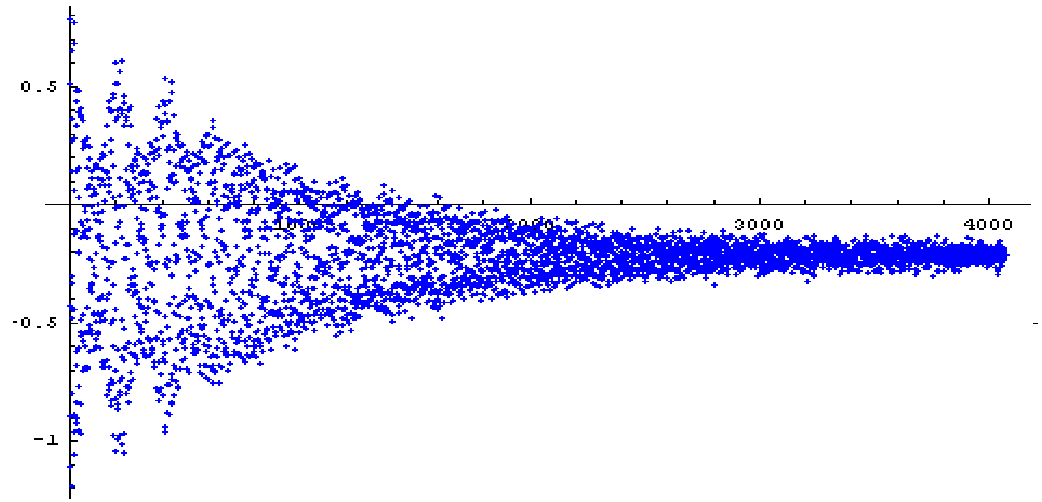


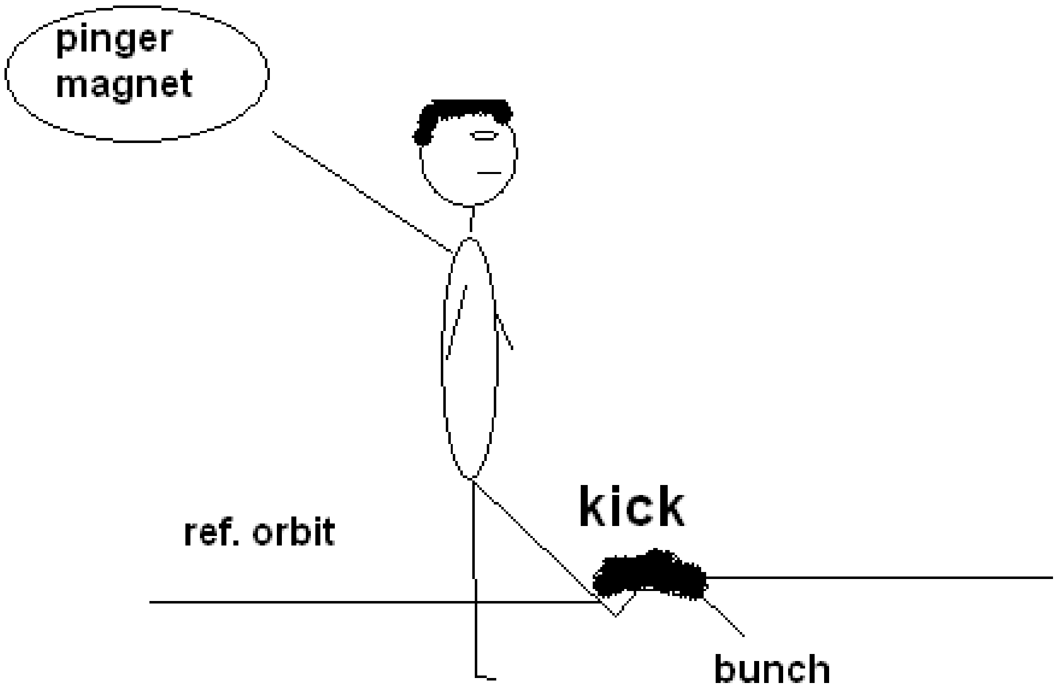
Comparision

Theoretical



Experimental

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magnet
kick
ref. orbit
bunch