X-RAY FOCUSING BY THE SYSTEM OF REFRACTIVE LENS(ES) PLACED INSIDE ASYMMETRIC CHANNEL-CUT CRYSTALS (RDL)

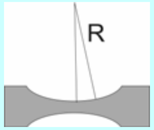
Arshak H. Grigoryan, Minas K. Balyan, Albert H. Toneyan

Grigoryan,Balyan, Toneyan,
Journal of Synchrotron Radiation, (2010), 17, 332-347.

This report is devoted to a new method of focusing X-rays.

1.Formulation of the Problem

Single lens focal distance



$$F_0 = \frac{R}{2\delta}; \quad (1)$$ **decrement** $\delta = 1 - n > 0;$ R **Curvature radius**

For X-rays $\lambda \sim 1A; \quad \delta \sim 10^{-5} - 10^{-6}; R \sim 1mm; F_0 \approx 10^2 - 10^3 m$

The focal distances of X-ray lenses are very large (refractive index is very close to 1), and their application meets principle difficulties. It becomes essential to have schemes, which have focusing distances (of 1m order) appropriate for application. Recently an X-ray focusing system has been suggested **(Grigoryan *et al.*, 2004;2010)** that consists of (+n,-n) asymmetric reflecting two plane parallel crystal plates which are considered to be half-infinite and the double-concave cylindrical parabolic lens placed in the gap between the plates (RDL) (Fig. 1).

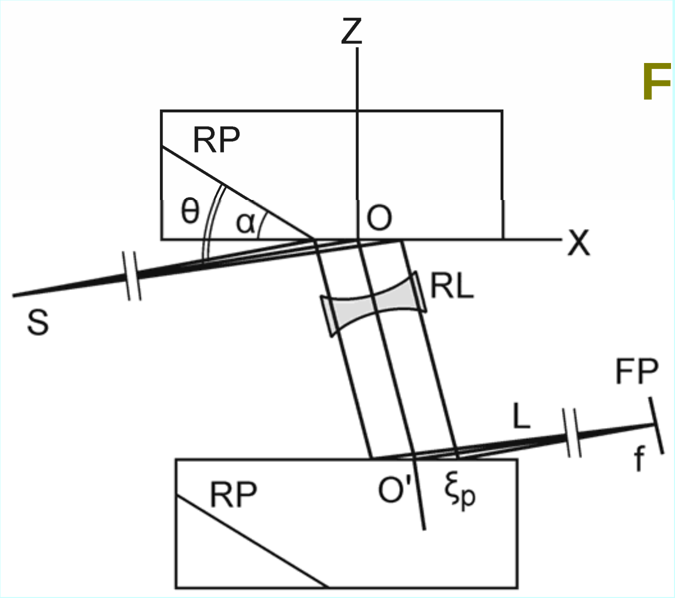


Fig.1 GENERAL FOCUSING SCHEME OF RDL.

RP-reflecting planes,
θ–incident angle, α–asymmetry angle,
RL-Refractive lens,FP-focusing plane,
f-focus, L-distance RDL-observation point, ξ_p-coordinate across double reflected beam.

2.FOCAL DISTANCE, LENS FORMULA of RDL

Distance source-crystal L_s

Effective distance for reflected beam

$$L_s^* = L_s / b^2; \quad (2)$$

Asymmetry factor $b = \sin(\theta - \alpha) / \sin(\theta + \alpha);$

Lens formula in the gap $1/L_s^* + 1/L_{hf} = 1/F_0;$ (3)

After second reflection $L_s^* b^2 = L_s; L_f = L_{hf} b^2;$

Focal Distance of RDL $F = F_0 b^2;$ (4)

Lens Formula of RDL $1/L_s + 1/L_f = 1/F;$ (5)

Magnification $M = L_f / L_s.$

All wavelengths are focused in the same point, there is no chromatic aberrations

Focus Spot Size $$\Delta\xi_{pf} \sim \frac{\gamma_0 \sin 2\theta_0}{k|\chi_h|\sqrt{\gamma_0\gamma_h}} = \frac{\sqrt{\gamma_0\gamma_h}}{k|\chi_{hr}|} \sim 10^{-4}cm \qquad (6)$$

Intensity at focus spot $$I_f(0,F) = I_0 \frac{\pi F k|\chi_{hr}|^2\gamma_h}{2\sin^2 2\theta_0\gamma_0} \sim 10^2 I_0 \qquad (7)$$

Examples

Si(220)MoKa radiation $\lambda = 0.701A$ Bragg angle $\theta_0 = 10.626^\circ$

Asymmetry parameters $\gamma_0 = 0.0175,\ \gamma_h = 0.346, \quad b = 0.05$

Asymmetry angle $\alpha = 9.626^\circ$

1. Be Lens (non absorptive lens) $\mu T_{\max} = 0.05$

Geometrical parameters of single lens $R = 1mm$ $r_0 = 0.1mm$ **Aperture 2mm**

Focal distance for single Be lens

$$F_0 = \frac{R}{2\delta} = 448.43m$$

Focal distance of RDL by Be lens

$$F = 1.140m$$

2. Si Lens (absorptive lens) $\mu T_{\max} = 1.5$

Focal distance for single Si lens

$$F_0 = \frac{R}{2\delta} = 316.26m$$

Focal distance of RDL with Si lens

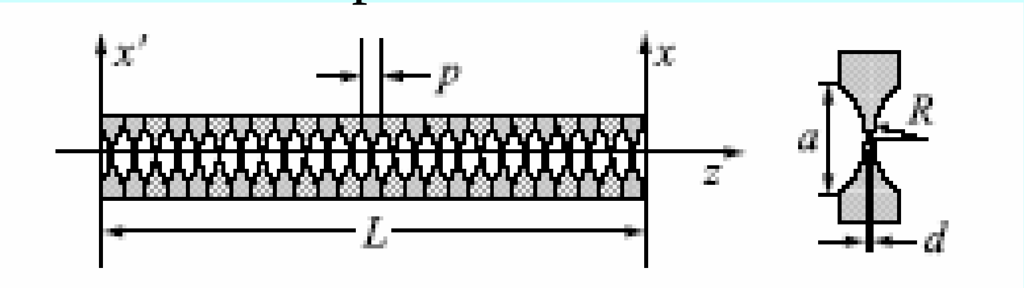
$$F = 0.804m$$

For compound lens $F_{comp} = F_0 / N;$ N **is number of lenses in array**

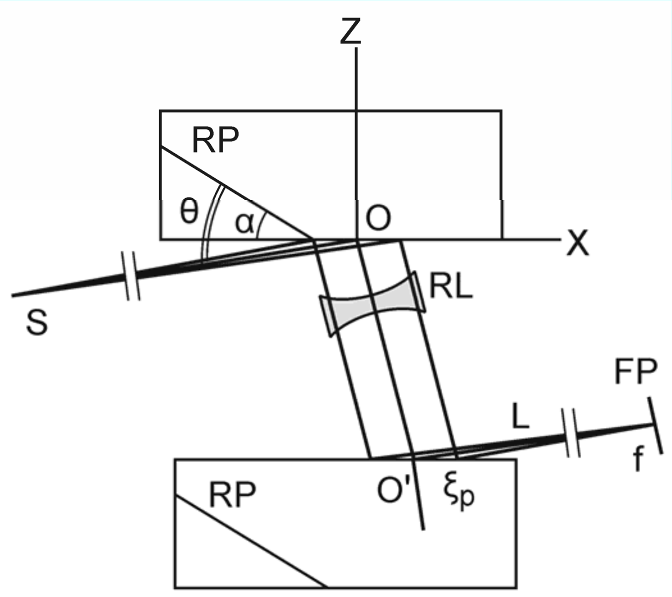
RDL is equivalent a compound lens with N=1/b^2

For taken example b=0.05, N=400; i.e. RDL is equivalent to a compound lens with 400 lenses

Compound Lens



RDL



Kohn V.G.,JETP, Letters,(2002),76,10,701-704.

3. Transmission

$$\Sigma_t(\Delta\lambda, L_s) = \exp(-2k|\beta|T_0)\int_{-R_{0x}}^{R_{0x}} |\Gamma|^2 \exp(-k\gamma_1 x^2 \gamma_0^2 / F)\,dx/(2R_{0x})$$

$$\Gamma(x,\lambda) = \Gamma_1(x,\lambda)\Gamma_2(x,\lambda)$$ **Total Reflection coefficient**

Reflection coefficient for first plate

$$\Gamma_1(x,\lambda) = -\sqrt{\chi_h/\chi_{\bar{h}}}\sqrt{\gamma_0/\gamma_h}\,\sigma/[k\Delta\theta(x,\lambda)\gamma_0 + \sigma_0 + \sqrt{(k\Delta\theta(x,\lambda)\gamma_0 + \sigma_0)^2 - \sigma^2}\,]$$

Reflection coefficient for second plate

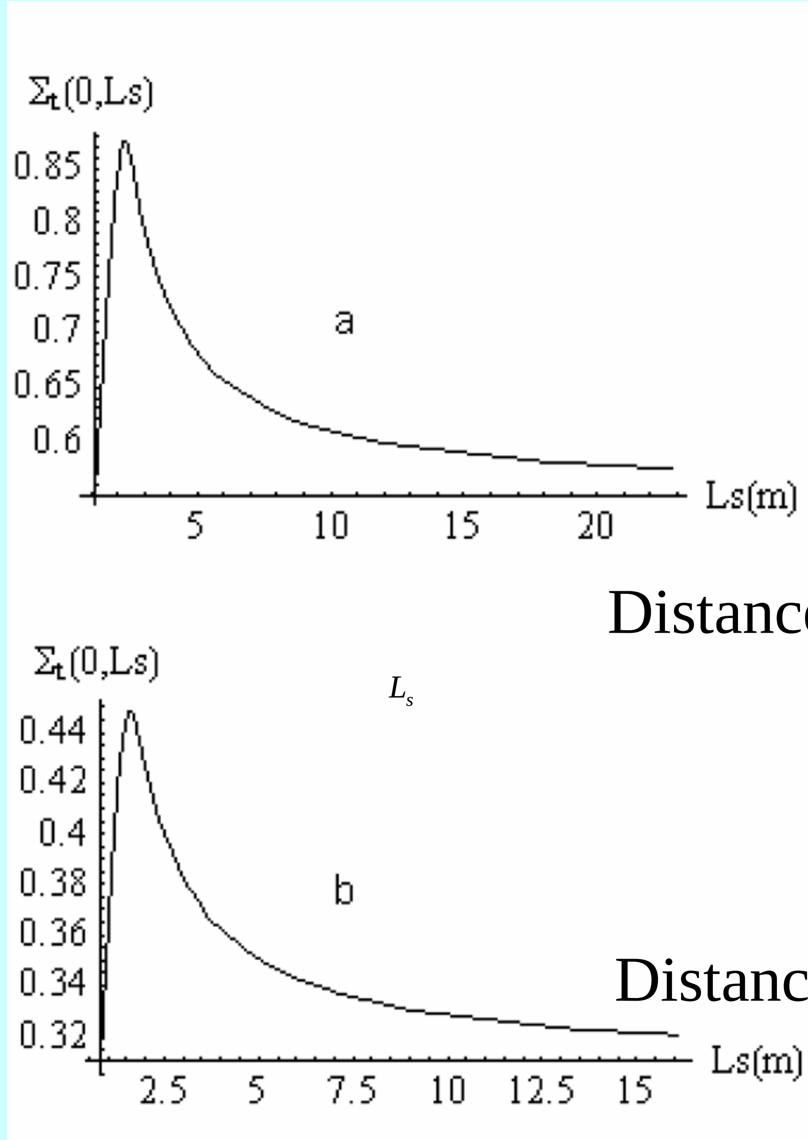
$$\Gamma_2(x,\lambda) = -\sqrt{\chi_{\bar{h}}/\chi_h}\sqrt{\gamma_h/\gamma_0}\,\sigma/[k\Delta\theta'_h(x,\lambda)\gamma_h + \sigma_0 + \sqrt{(k\Delta\theta'_h(x,\lambda)\gamma_h + \sigma_0)^2 - \sigma^2}\,]$$

Deviation from Bragg exact angle, influence of the lens On Bragg condition

$$\Delta\theta'_h(x,\lambda) = b\Delta\theta(0,\lambda) - bx\gamma_0(1/L_s - 1/F)$$

Transmission
Be Lens

Transmission
Si Lens

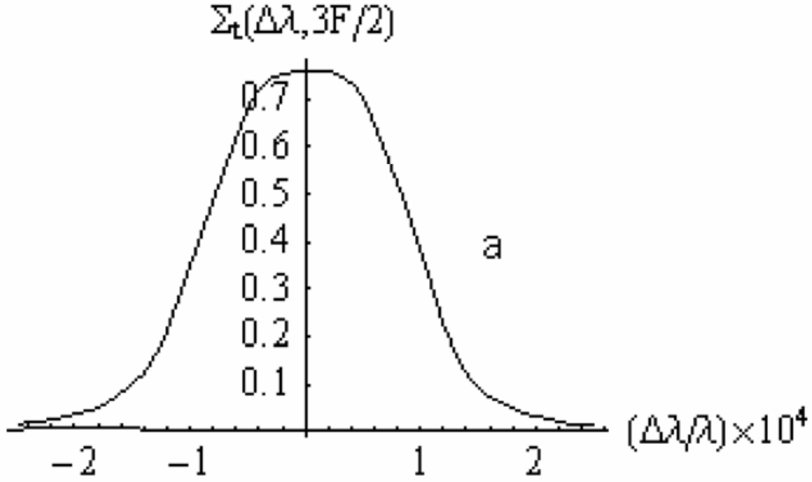


Distance source-RDL Ls(m)

Distance source-RDL Ls(m)

Fig. 6 The RDL 's transmission dependence on L_s for $\Delta\lambda = 0$
(a)-Lens made by beryllium, (b)- lens made by silicon

Transmission Be Lens



Throw RDL effectively pass waves with wavelengths

$$\Delta\lambda / \lambda = 10^{-4}$$

Transmission Si Lens

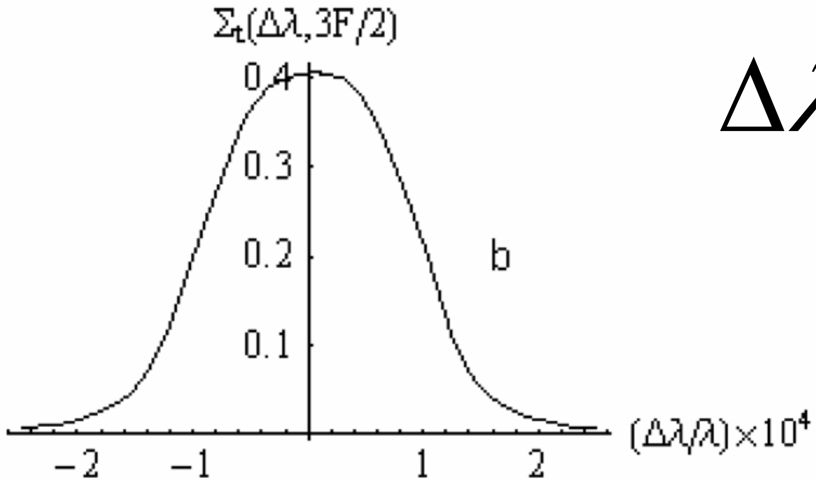


Figure 7 The RDL's transmission dependence on $\Delta\lambda$ for Ls=3F/2. (a)-Lens made by beryllium, (b)- lens made by silicon.

4. RESOLUTION

Angular resolution

$$\left|\xi_{pf\,1} - \xi_{pf\,2}\right| = F\left|\theta_1 - \theta_2\right| = 3.8\gamma_0 / \sigma_r$$

$$\left|\theta_1 - \theta_2\right| \cong 1.83\times 10^{-6} = 0.37''$$

Spatial Resolution

$$\Delta\xi_s = \Delta\xi_f L_s / L_f = 3.8[(1+M)/M]\gamma_0 / \sigma_r$$

$$\Delta\xi_s = 1.8\mu m$$

$$\Delta x_s = \Delta x_f L_s / L_f = \pi L_s / kR_0$$

$$\Delta x_s \geq 15\mu m$$

5. Conclusions

1. A 1D focusing X-ray element is proposed. This element includes two plane parallel asymmetric cut crystalline plates and a cylindrical parabolic double-concave lens placed in the gap between the plates.
2. It is shown, that the focal distance of this element is equal to the focal distance of separate taken lens multiplying by the square of the asymmetry factor $F = F_0 b^2$
3. RDL is equivalent to an X-ray compound lens with lenses $N=1/b^2$. For example, when $F_0=400$m, $b=0.05$, then $F=1$m and $N=400$.
4. In paraxial approximation the focus point does not depend on wavelength.RDL is achromatic lens.
5. Focus size is 0.1-1mkm. Angular resolution 0.37”, spatial resolution 0.1-1mkm.
6. The intensity in the point of observation can increase by two orders (10^2).
7. The transmission of RDL is close to 1. For a compound lens, the transmission can be up to 0.3.
8. Geometrical parameters: *R* up to 1 cm, geometrical aperture *up to* 1cm, material of lens: *Be*, *Al*, *Si*…
9. RDL can be upgraded to two dimensional focusing scheme by using two RDLs with perpendicular planes of diffraction.
10. RDL can focus both laboratory and synchrotron X-ray sources.
11. RDL can be used in X-ray astronomy, for imaging of objects, in X-ray microscopy. RDL can be used as collimator, if a point source is placed at the distance *F* from RDL.

Thank You For Your Attention