Introduction to GRIL Code: Wakefields and Impedance in Multilayer Chamber

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Problem Description

Circular-cylindrical many-layer tube

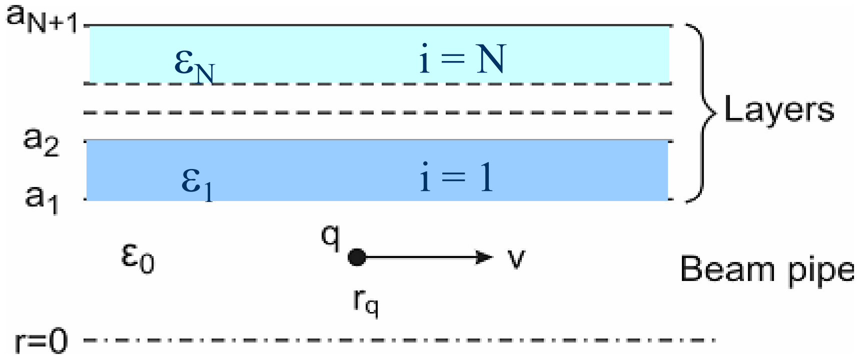


Fig.1. Geometry of the laminated vacuum chamber.

$a_{1,}$ - **inner radius**

a_{N+1} - **outer raius**

ε_i - **permittivity**

μ_i – **magnetic permeability**

(i=1,2,3,…N)

q - **point-like charge**

r_q - **offset**

v - **constant velocity**

Solution Method

The impedance of multi-layer vacuum chamber

walls can be found by solving Maxwell’s equations

taking into account the proper boundary conditions.

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

N layer tube → { N+1 borders; N+2 regions }

4 boundary equations for each border that matches E_θ, E_z, B_θ, B_z, components of E&M field

↓

4(N+1) equations, 4(N+1) unknowns

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1N}x_N &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2N}x_N &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3N}x_N &= b_3 \\
\cdots \qquad & \quad \cdots \\
a_{M1}x_1 + a_{M2}x_2 + a_{M3}x_3 + \cdots + a_{MN}x_N &= b_M
\end{aligned}
$$

Solving these equations gives the electric field, which then gives the impedance.

Fields Transformation Matrix

For each ith layer:

$$\hat{Q}^{(i)} = \begin{pmatrix} q_{11} & q_{12} & -\alpha_0 q_{31} & \alpha_0 q_{32} \\ 0 & q_{22} & \alpha_0 q_{41} & -\alpha_0 q_{42} \\ q_{31} & q_{32} & q_{11} & -q_{12} \\ q_{41} & q_{42} & 0 & q_{22} \end{pmatrix}$$

$$q_{ij} \Rightarrow \begin{matrix} \varepsilon_i, \mu_i, \\ a_i, a_{i+1} \end{matrix}$$

$T_{in} = Q\,T_{out}$

$Q = Q_1 \cdot Q_2 \cdot \ldots \cdot Q_N$

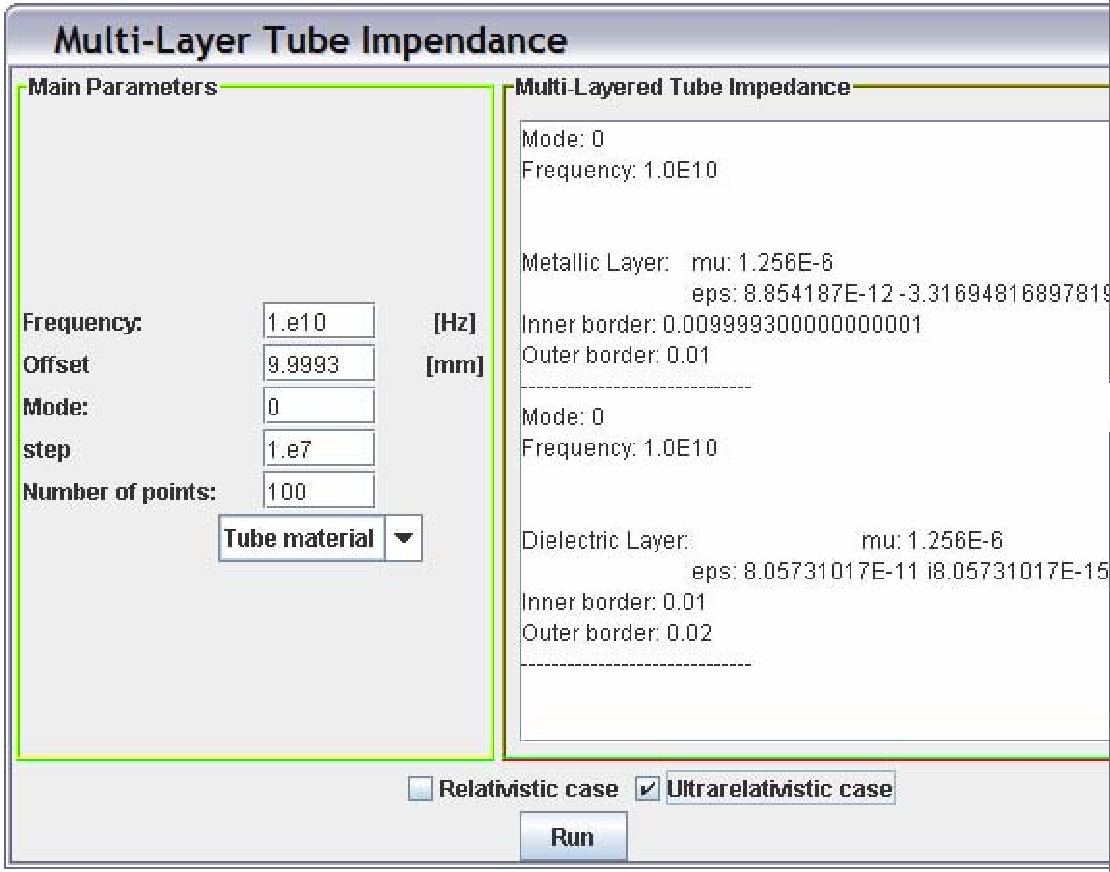
T_{in} –vector of tangential E&M fields in inner surface .

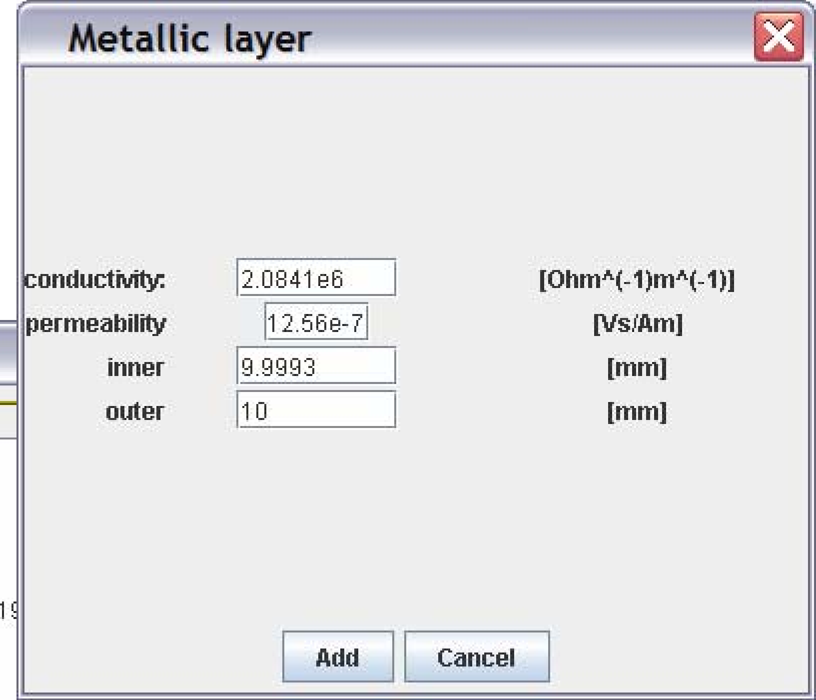
T_{out}- vector of tangential E&M fields in outer surface

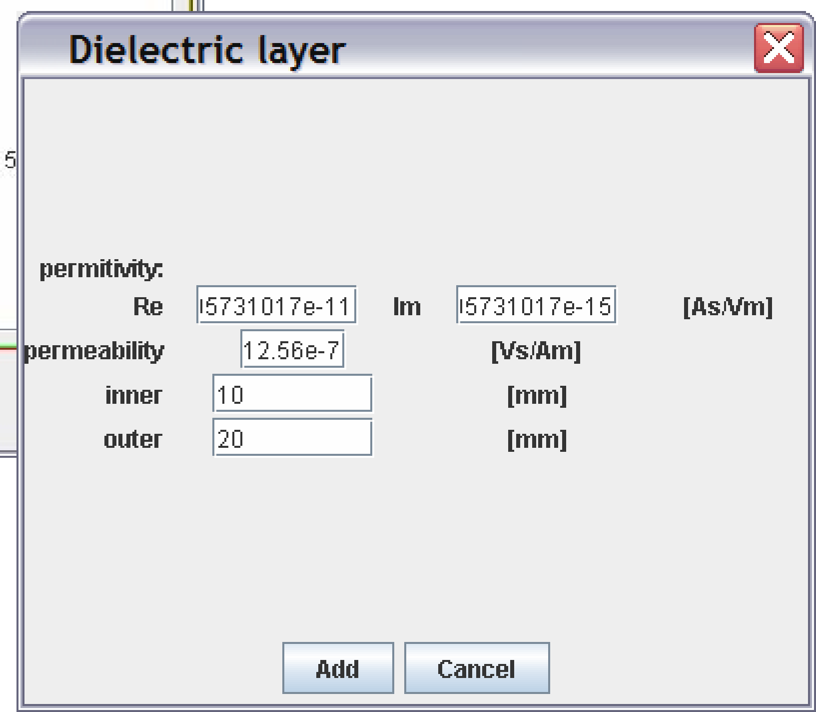
Q (4x4)– Field Transformation Matrix of Laminated Tube

** M. Ivanyan et.all, Multilayer Tube Impedance and External Radiation, Physical Review Special Topics – Accelerators and Beams*

GRIL Code


Multi-Layer Tube Impendance
Main Parameters
Frequency: 1.e10 [Hz]
Offset 9.9993 [mm]
Mode: 0
step 1.e7
Number of points: 100
Tube material
Multi-Layered Tube Impedance
Mode: 0
Frequency: 1.0E10
Metallic Layer: mu: 1.256E-6
eps: 8.854187E-12 -3.31694816897819
Inner border: 0.009999300000000001
Outer border: 0.01
Mode: 0
Frequency: 1.0E10
Dielectric Layer: mu: 1.256E-6
eps: 8.05731017E-11 i8.05731017E-15
Inner border: 0.01
Outer border: 0.02
Relativistic case
Ultrarelativistic case
Run


Metallic layer
conductivity: 2.0841e6 [Ohm^(-1)m^(-1)]
permeability 12.56e-7 [Vs/Am]
inner 9.9993 [mm]
outer 10 [mm]
Add
Cancel


Dielectric layer
permitivity:
Re i5731017e-11 Im i5731017e-15 [As/Vm]
permeability 12.56e-7 [Vs/Am]
inner 10 [mm]
outer 20 [mm]
Add
Cancel

GRIL Code output