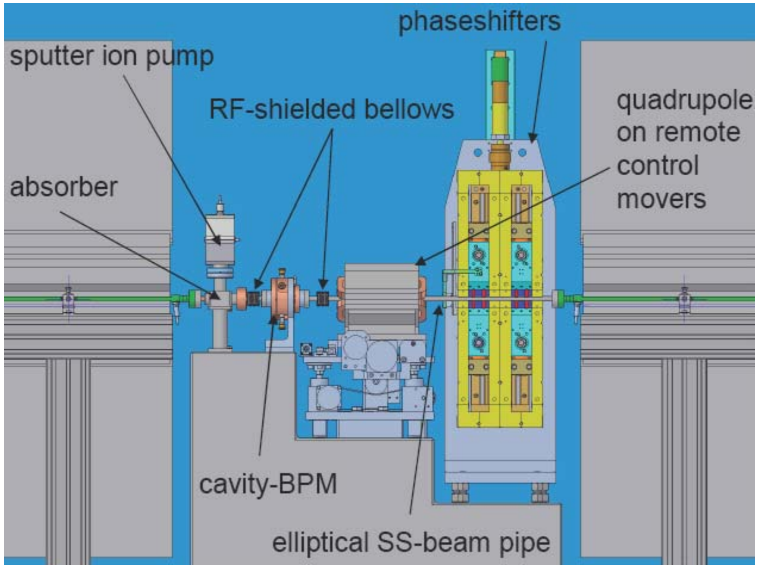
Topics

- Introduction
- Formulation of the problem
- Physical motivation of the model
- Algorithm description
- Numerical tests with analytics and CST-PS
- Practical application
- Summary

Introduction



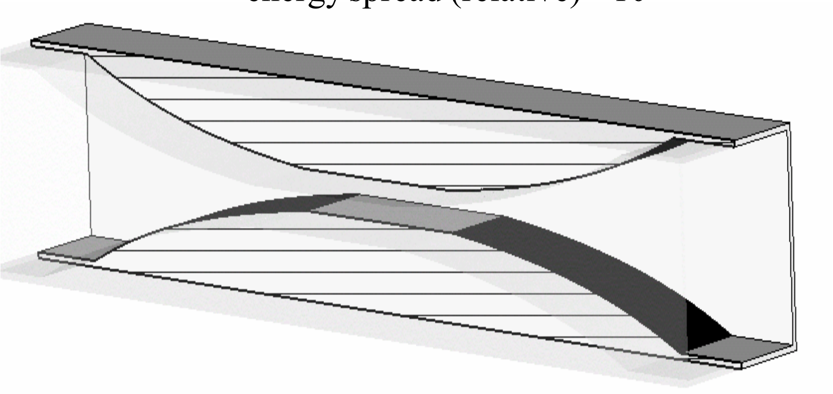
Transient Wake Fields in XFEL Undulator Intersection

The Bunch parameters which is required in European XFEL project

r.m.s. bunch lenght ~ 25 μm

emittance ~ 1.4 mm · mrad

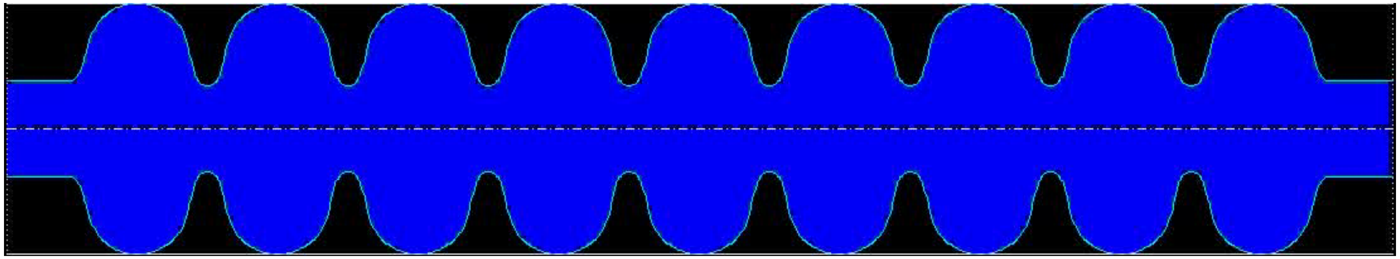
energy spread (relative) ~ 10^{-4}



Geometrical and Resistive Wake Fields in Collimator

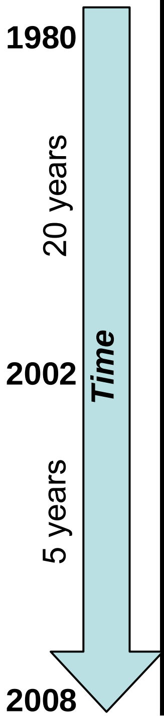
Wake Fields

Geometric + Resistive = ?



Introduction

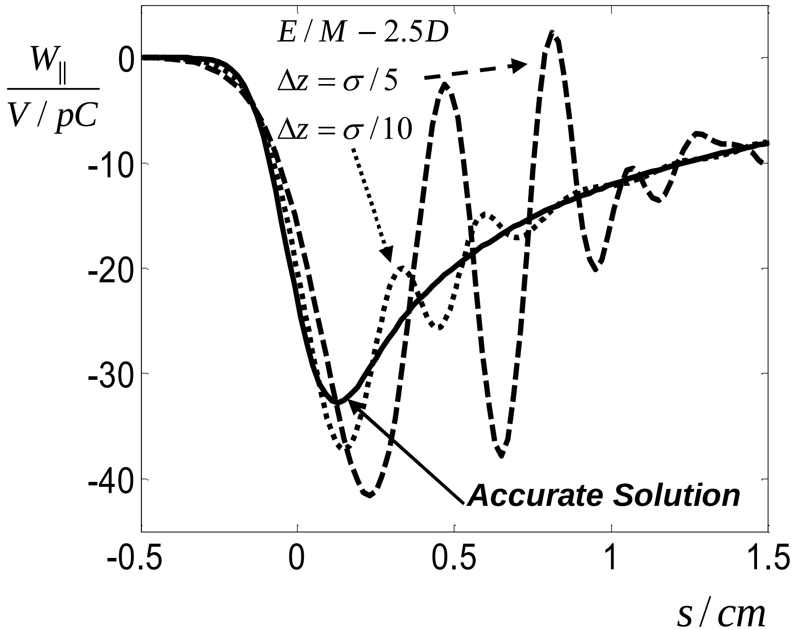
An (incomplete) survey of available codes

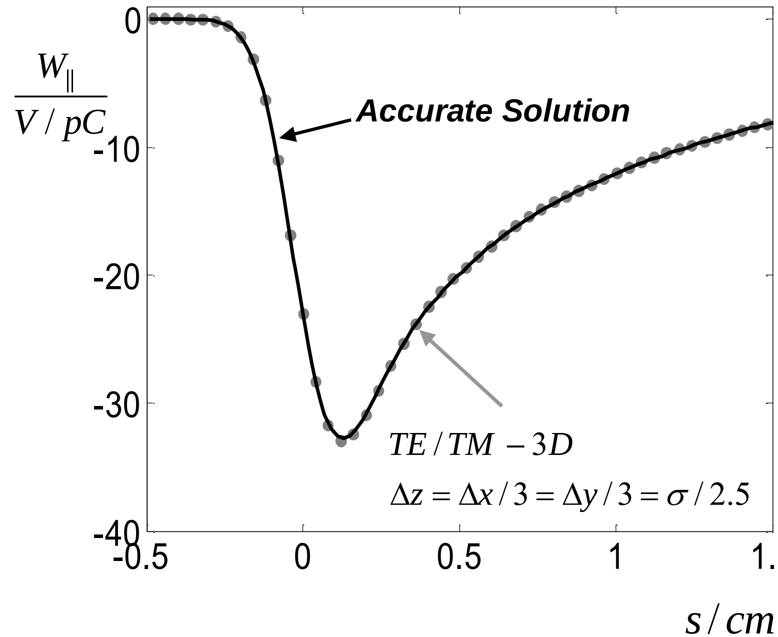


	Non-dispersive in longitudinal direction	Second order convergence	Conductivity
BCI/TBCI	No	No	No
NOVO	Yes	No	No
ABCI	No	No	No
MAFIA	No	No	No
XWAKE	No	Yes	No
Gdfidl	No	No	No
Tau3P	No	Yes	No
ECHO	Yes	Yes	No
CST	No	Yes	Yes
PBCI	Yes	No	No
NEKCEM	No	Yes	No

Introduction

$\sigma = 1mm$



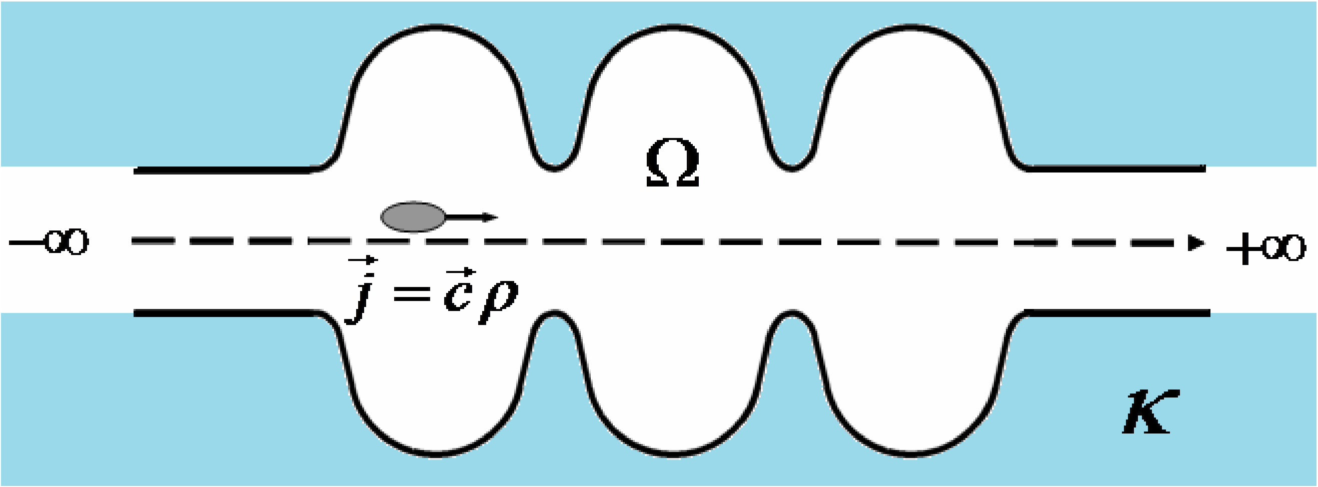


Comparison of the wake potentials obtained by different methods for structure consisting of 20 TESLA cells excited by Gaussian bunch

E/M splitting	TE/TM splitting
$\Delta z << \sqrt{\frac{\sigma^3}{L}}$	$\Delta z < \sigma$

Conditions for longitudinal dispersion damping

Formulation of the Problem



Ultra relativistic charged particle moving through an accelerating structure with finite conductive walls supplied with infinite pipes.

$$Curl\ \vec{E} = -\frac{\partial}{\partial t}\mu\vec{H} \qquad Div\ \varepsilon\vec{E} = \rho$$

$$Curl\ \vec{H} = \vec{J} + \frac{\partial}{\partial t}\varepsilon\vec{E} \qquad Div\ \vec{H} = 0$$

$$\lambda_{metal} = \frac{2\pi}{\sqrt{\omega\mu\sigma}}$$

$$\lambda_{vacuum} = \frac{2\pi c}{\omega}$$

Stainless Steel

$$\kappa = 1.4 \cdot 10^6 \ \Omega^{-1} m^{-1}$$

$$\sigma = 25\,\mu m \Rightarrow \frac{\lambda_{vacuum}}{\lambda_{metal}} \approx 80$$

$$\sigma = 1mm \Rightarrow \frac{\lambda_{vacuum}}{\lambda_{metal}} \approx 500$$

Copper

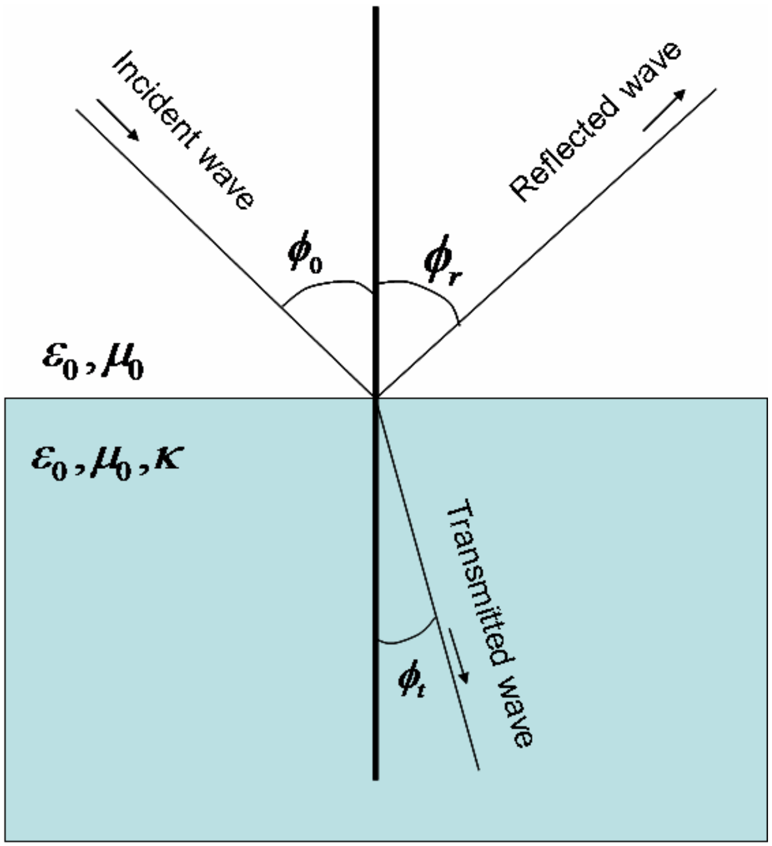
$$\kappa = 58 \cdot 10^6 \ \Omega^{-1} m^{-1}$$

$$\sigma = 25\,\mu m \Rightarrow \frac{\lambda_{vacuum}}{\lambda_{metal}} \approx 500$$

$$\sigma = 1mm \Rightarrow \frac{\lambda_{vacuum}}{\lambda_{metal}} \approx 3000$$

Physical motivation of the model

Transmission of EM wave on vacuum-conductor boundary surface.



$$\sin\phi_t = \frac{1}{n(\phi_0, \omega, \kappa)} \sin\phi_0$$

$$\kappa >> \varepsilon_0 \omega \quad \Rightarrow \quad \phi_t \sim 0$$

Example

Stainless Steel - $\kappa = 1.4 \cdot 10^6\, \Omega^{-1} m^{-1}$

r.m.s bunch lenght - 25 μm

$$\kappa / \varepsilon_0 \omega \sim 10^4$$

Algorithm Description

Based on finite integration technique (FIT) discretization

$$\oint_{\partial S} \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\iint_S \mu\vec{H}\cdot d\vec{A}$$

$$\oint_{\partial S}\vec{H}\cdot d\vec{l} = \iint_S\left[\vec{J}+\frac{\partial}{\partial t}\varepsilon\vec{E}\right]\cdot d\vec{A}$$

$$\iint_S \varepsilon\vec{E}\cdot d\vec{A} = \iiint_V \rho\, dV$$

$$\iint_S \mu\vec{H}\cdot d\vec{A} = 0$$

FIT

$$C\widehat{e} = -\frac{d}{dt}M_\mu \widehat{h}$$

$$C^T\widehat{h} = \widehat{\widehat{j}} + \frac{d}{dt}M_\varepsilon\widehat{e}$$

$$S M_\varepsilon \widehat{e} = q$$

$$S^T M_\mu \widehat{h} = 0$$

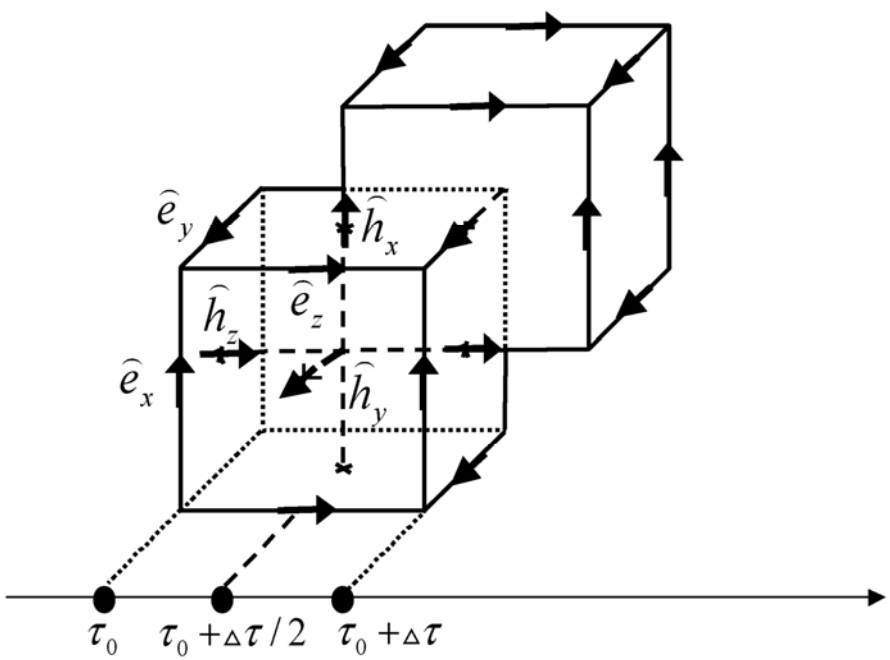
~

$$\mathrm{Curl}\ \vec{E} = -\frac{\partial}{\partial t}\mu\vec{H}$$

$$\mathrm{Curl}\ \vec{H} = \vec{J} + \frac{\partial}{\partial t}\varepsilon\vec{E}$$

$$\mathrm{Div}\ \varepsilon\vec{E} = \rho$$

$$\mathrm{Div}\ \vec{H} = 0$$



C ~ Curl

S ~ Div

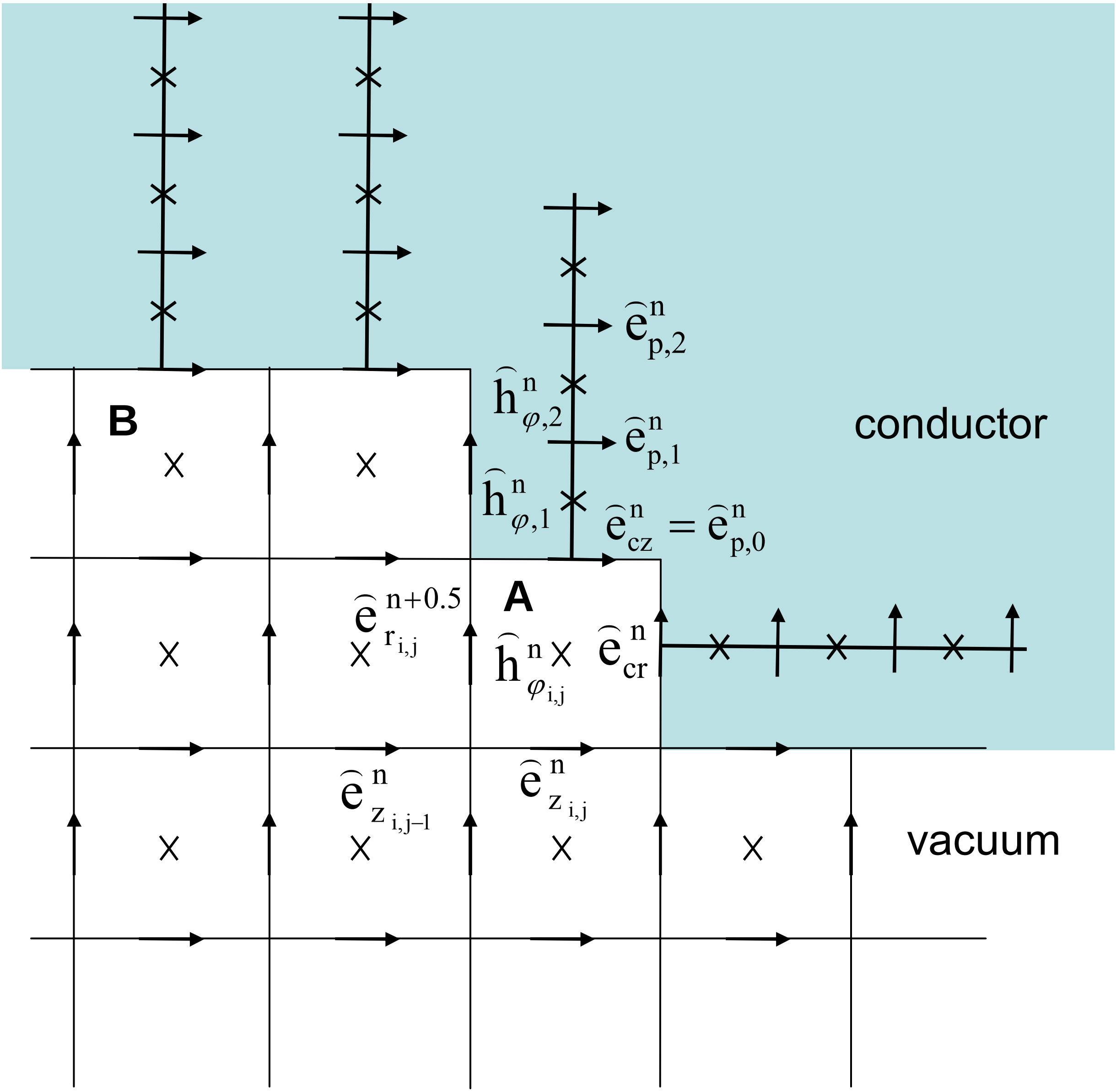
Basic properties of continuous operators are retained

$$S^T C = S C^T = 0 \quad \leftrightarrow \quad div\, curl = 0$$

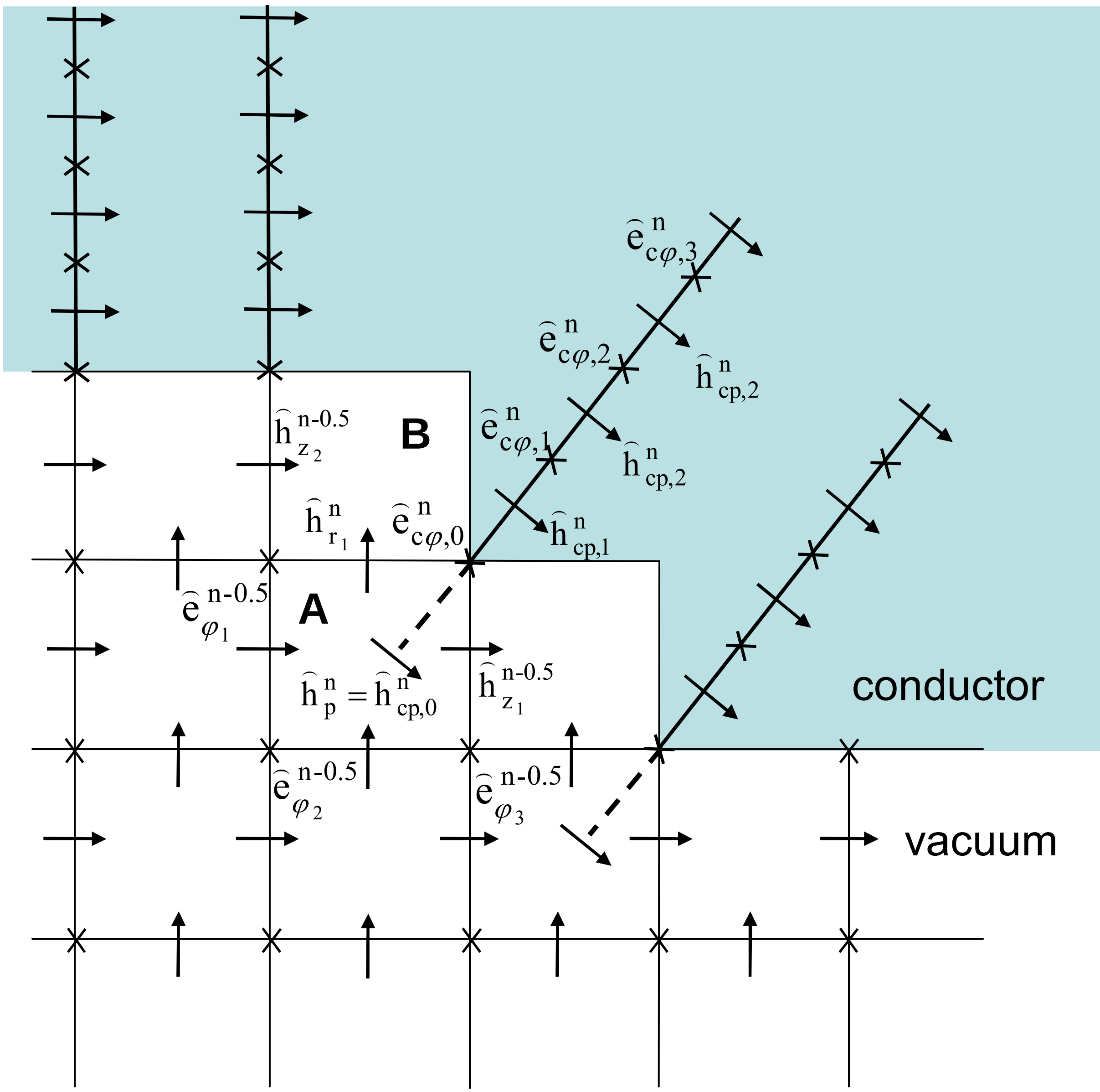
Finite Integration Technique

- T. Weiland (1978)

Vacuum grid with 1D conducting lines at the boundary for monopole case (m=0)



Vacuum grid with 1D conducting lines at the boundary for m-th multipole case



Longitudinal dispersion free hybrid TE/TM numerical scheme

$$\widehat{h}_\varphi^{\#} = \widehat{h}_\varphi^{n-0.5} + \frac{\Delta\tau}{2} M_{\mu_\varphi^{-1}} \left[P_z \widehat{e}_r^{\,n} - P_r \widehat{e}_z^{\,n-0.5} + \widehat{e}_{\varphi c}^{\,n-0.5} \right]$$

$$\widehat{h}_r^{\#} = \widehat{h}_r^{n-0.5} + \frac{\Delta\tau}{2} M_{\mu_r^{-1}} \left[-P_z \widehat{e}_\varphi^{\,n} + m \widehat{e}_z^{\,n-0.5} + \widehat{e}_{rc}^{\,n-0.5} \right]$$

$$\widehat{e}_z^{\,n+0.5} = \widehat{e}_z^{\,n-0.5} + \left(I + \frac{\Delta\tau^2}{4} M_{\varepsilon_z^{-1}} P_r^* M_{\mu_\varphi^{-1}} P_r + \frac{\Delta\tau^2}{4} m^2 M_{\varepsilon_z^{-1}} M_{\mu_r^{-1}} \right)^{-1} \Delta\tau\, M_{\varepsilon_z^{-1}} \left[P_r^* \widehat{h}_\varphi^{\#} - m \widehat{h}_r^{\#} + \widehat{\widehat{j}}_z^{\,n} \right]$$

$$\widehat{h}_\varphi^{n+0.5} = \widehat{h}_\varphi^{\#} + \frac{\Delta\tau}{2} M_{\mu_\varphi^{-1}} \left[P_z \widehat{e}_r^{\,n} - P_r \widehat{e}_z^{\,n+0.5} + \widehat{e}_{\varphi c}^{\,n+0.5} \right]$$

$$\widehat{h}_r^{n+0.5} = \widehat{h}_r^{\#} + \frac{\Delta\tau}{2} M_{\mu_r^{-1}} \left[-P_z \widehat{e}_\varphi^{\,n} + m \widehat{e}_z^{\,n+0.5} + \widehat{e}_{rc}^{\,n+0.5} \right]$$

Field update in conductor

Implicit 1D scheme

Algorithm Stability Condition

$$\Delta\tau \le \Delta z$$

$$\widehat{e}_\varphi^{\#} = \widehat{e}_\varphi^{\,n} + \frac{\Delta\tau}{2} M_{\varepsilon_\varphi^{-1}} \left[P_z^* \widehat{h}_r^{n+0.5} - P_r^* \widehat{h}_z^{n} + \widehat{\widehat{j}}_\varphi^{\,n+0.5} \right]$$

$$\widehat{e}_r^{\#} = \widehat{e}_r^{\,n} + \frac{\Delta\tau}{2} M_{\varepsilon_r^{-1}} \left[-P_z^* \widehat{h}_\varphi^{n+0.5} + m \widehat{h}_z^{n} + \widehat{\widehat{j}}_r^{\,n+0.5} \right]$$

$$\widehat{h}_z^{n+1} = \widehat{h}_z^{n} + \left(I + \frac{\Delta\tau^2}{4} M_{\mu_z^{-1}} P_r M_{\varepsilon_\varphi^{-1}} P_r^* + \frac{\Delta\tau^2}{4} m^2 M_{\mu_z^{-1}} M_{\varepsilon_r^{-1}} \right)^{-1} \Delta\tau\, M_{\mu_z^{-1}} \left[P_r \widehat{e}_\varphi^{\#} - m \widehat{e}_r^{\#} + \widehat{e}_{zc}^{\,n+0.5} \right]$$

$$\widehat{e}_\varphi^{\,n+1} = \widehat{e}_\varphi^{\#} + \frac{\Delta\tau}{2} M_{\varepsilon_\varphi^{-1}} \left[P_z^* \widehat{h}_r^{n+0.5} - P_r^* \widehat{h}_z^{n+1} + \widehat{\widehat{j}}_\varphi^{\,n+0.5} \right]$$

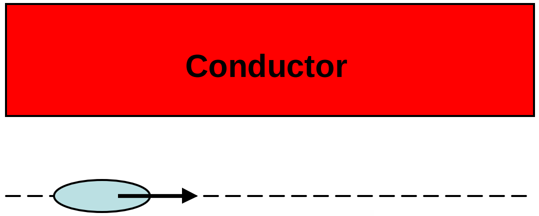
$$\widehat{e}_r^{\,n+1} = \widehat{e}_r^{\#} + \frac{\Delta\tau}{2} M_{\varepsilon_r^{-1}} \left[-P_z^* \widehat{h}_\varphi^{n+0.5} + m \widehat{h}_z^{n+1} + \widehat{\widehat{j}}_r^{\,n+0.5} \right]$$

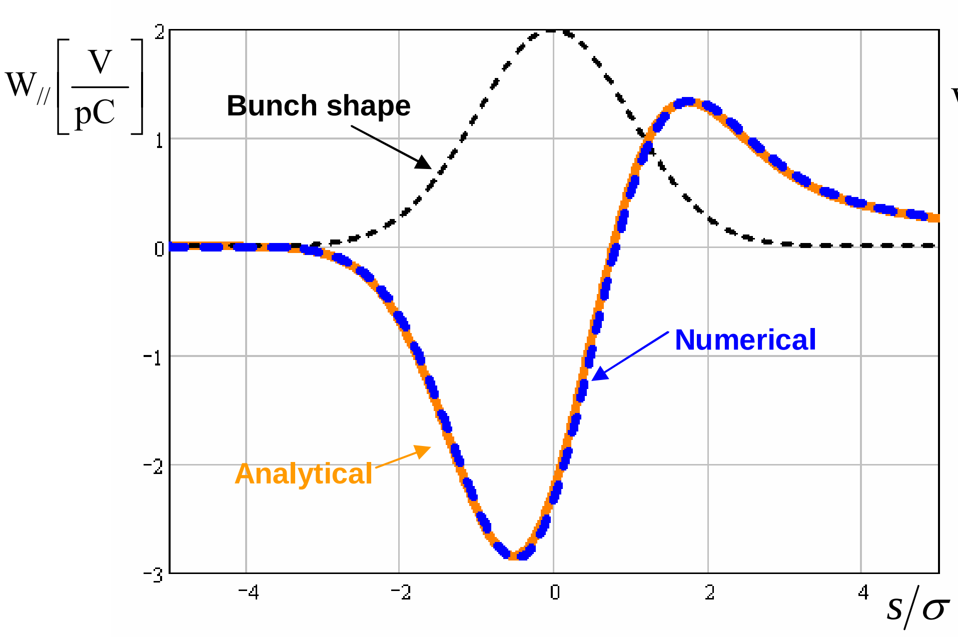
Longitudinal Dispersion Free Condition

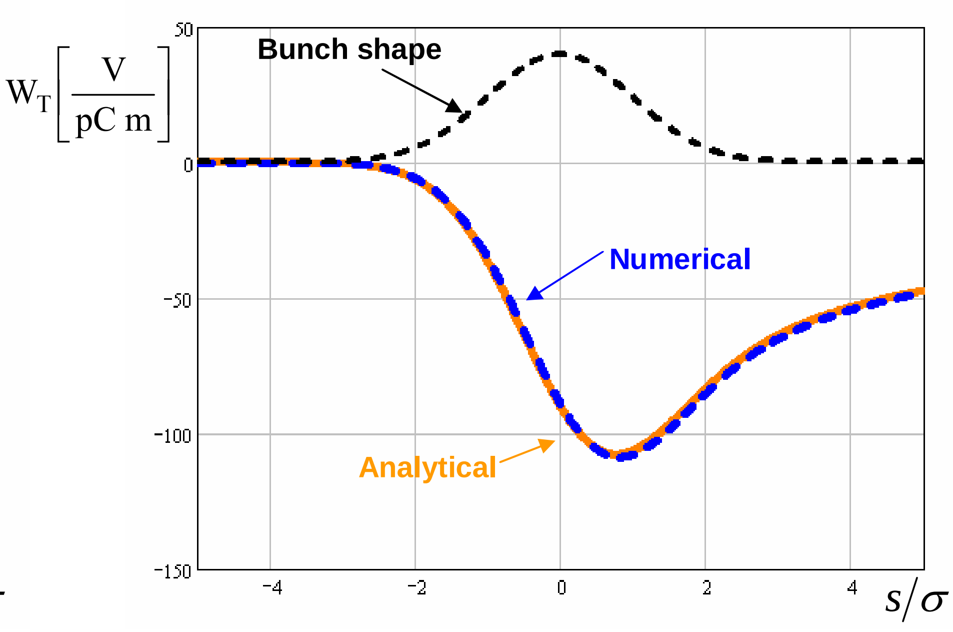
$$\Delta\tau = \Delta z$$

Comparison with analytic solutions

Comparison of numerical and analytical steady state wakes of the Gaussian bunch with rms length σ=1mm in round pipe of radius a=1 cm and of the conductivity κ=1e5 S/m and the.

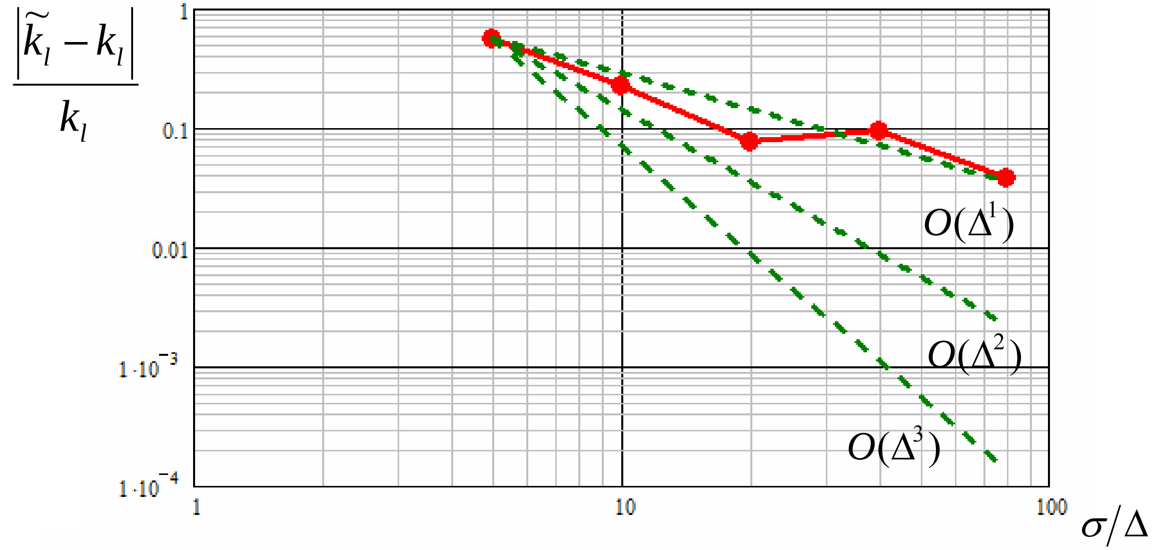


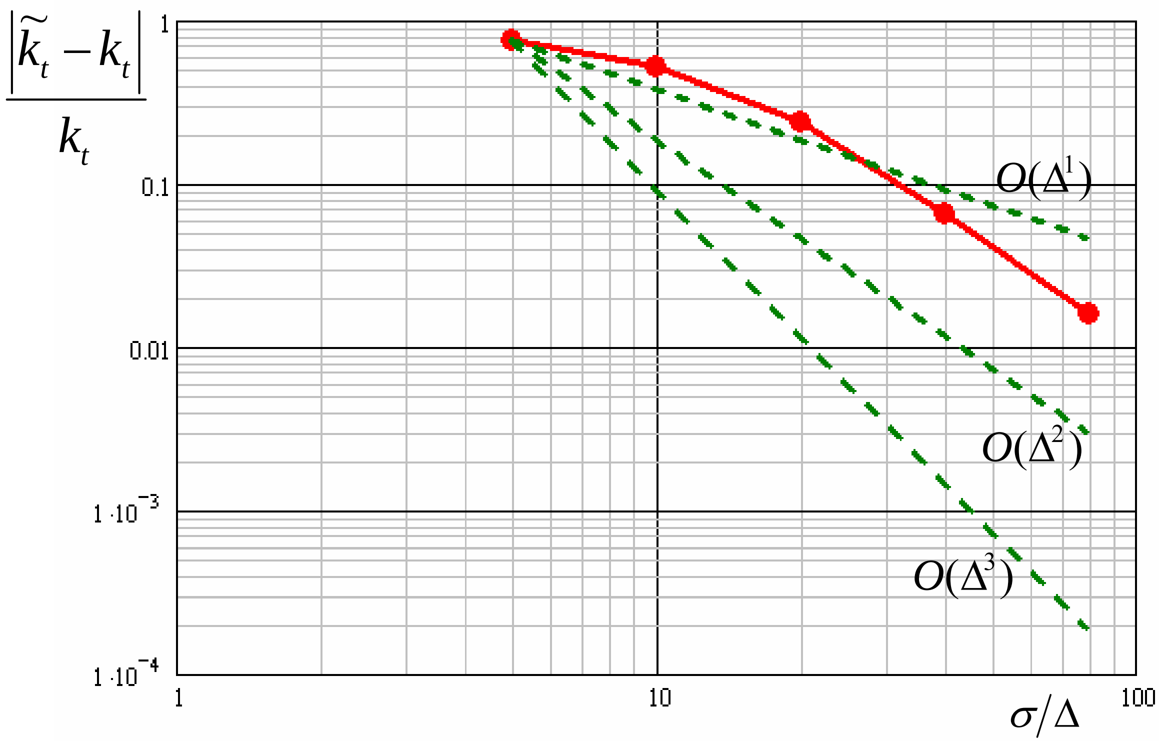




For mesh resolution of 10 points on σ error in loss factor is 3%

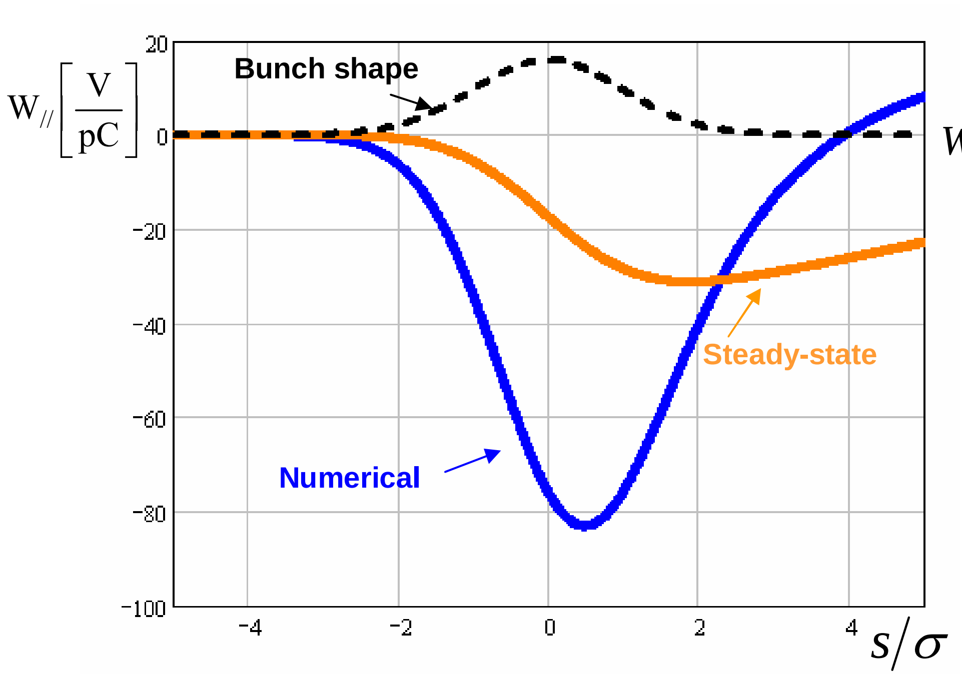
Convergence of the Scheme

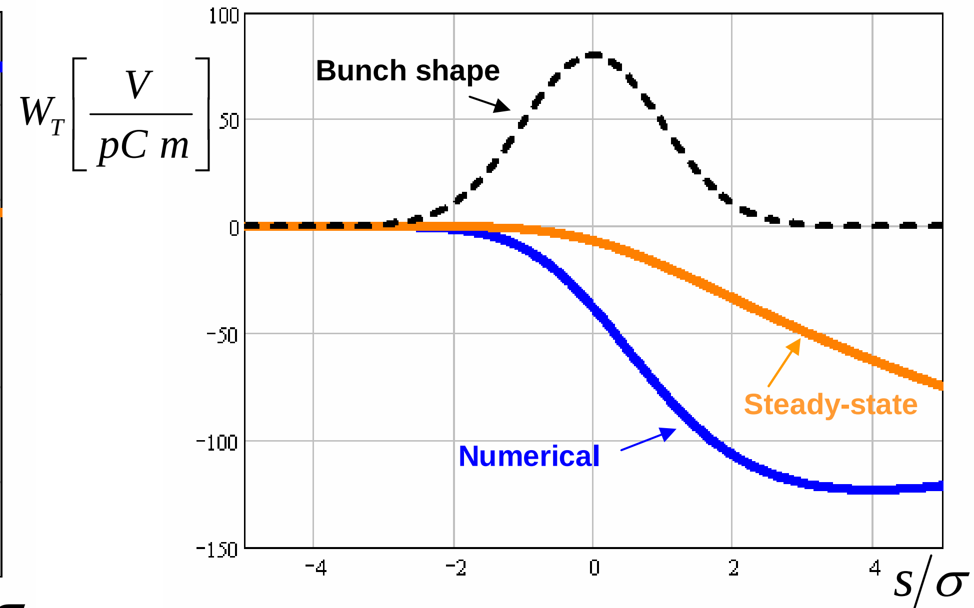




Comparison with analytic solutions

The wake potential of finite length resistive cylinder with radius a=1cm, length b=10cm and conductivity κ=1e4 S/m. The Gaussian bunch r.m.s. length is σ=25 μm.

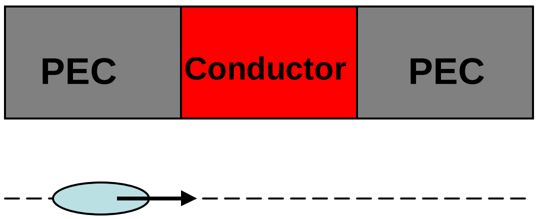




Loss Factor

Numerical = 58 V/pC

Analytic = 57 V/pC



Kick Factor

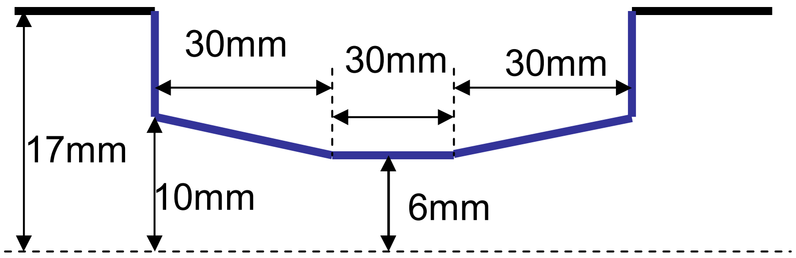
Numerical = 42.6 V/pC m^{-2}

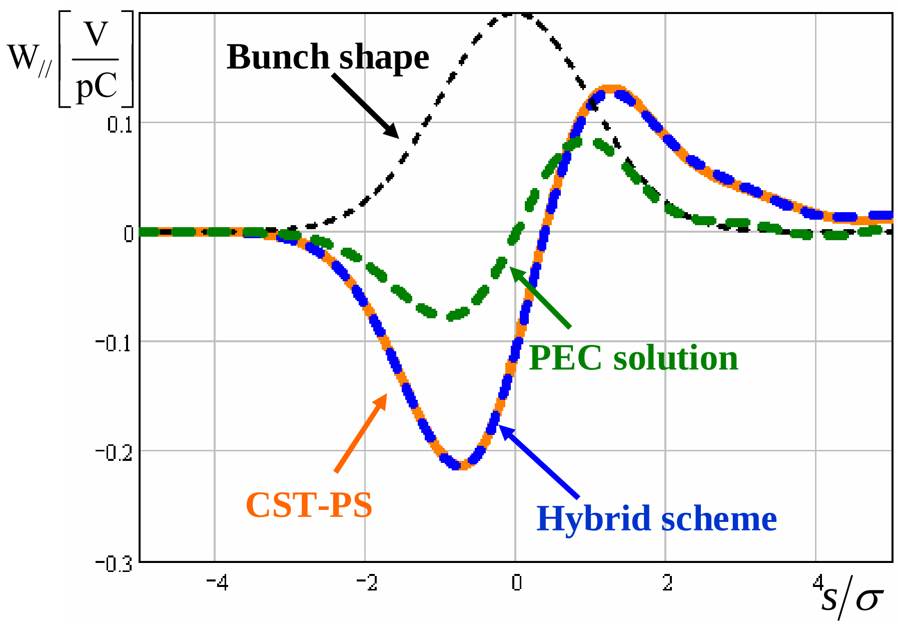
Analytic = 41.5 V/pC m^{-2}

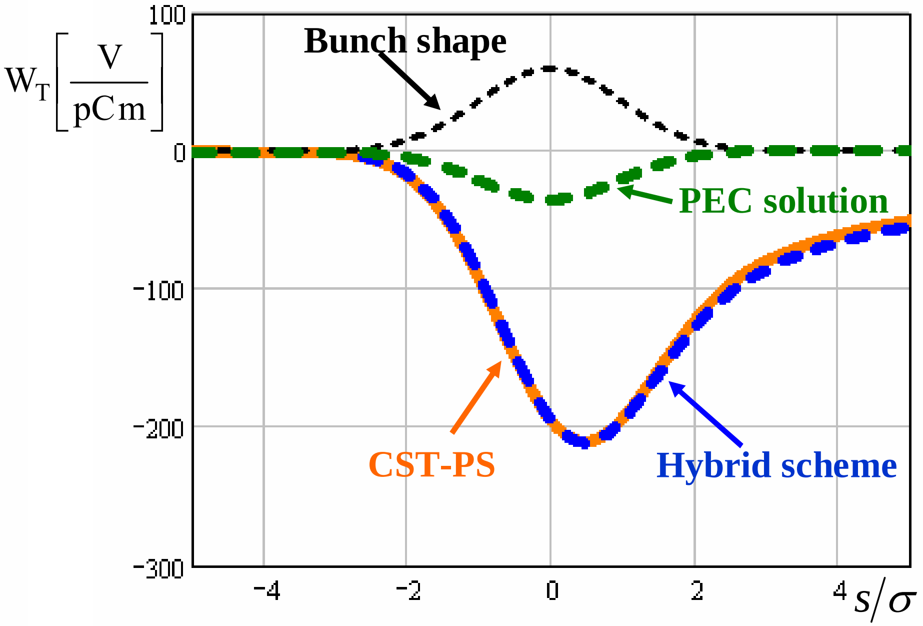
S.Krinsky and B. Podobedov, PR-STAB, 7, 114401 (2004)

Comparison with CST Microwave Strudio

Comparison of wake potentials of tapered collimator "with" and "without" resistivity
for Gaussian bunch σ = 20 mm .



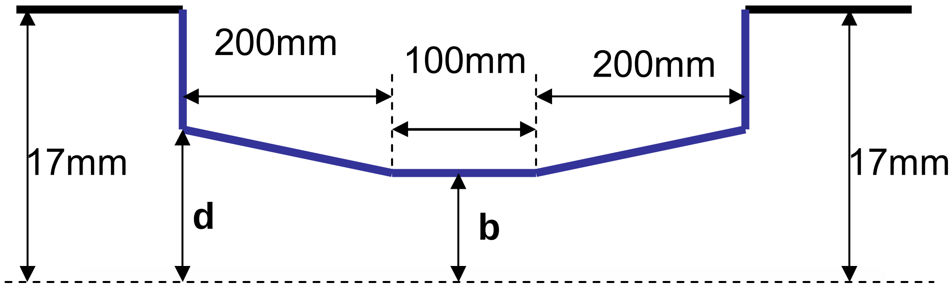




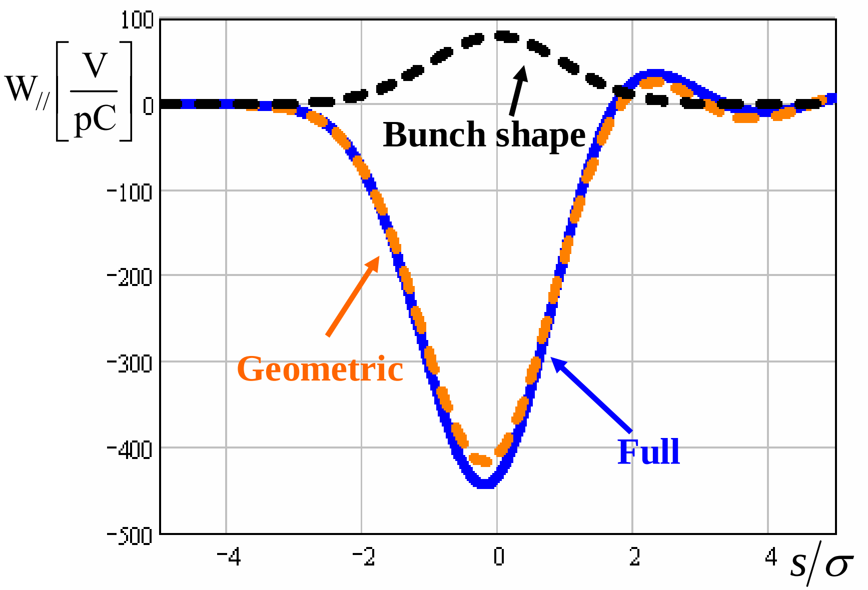
Loss factor for finite conductive walls cannot be obtain as direct sum of the geometrical and the steady-state solution.

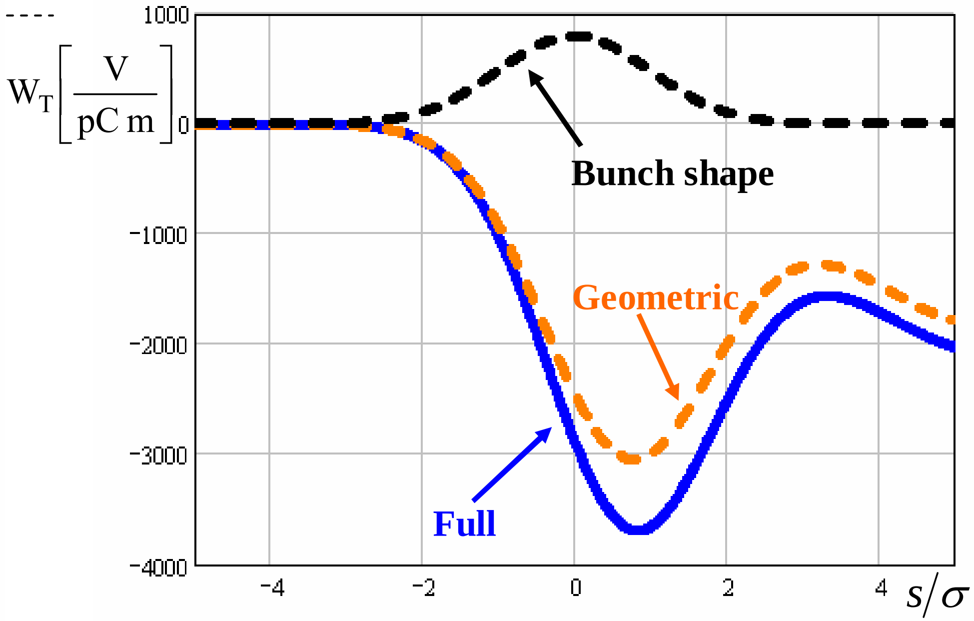
Practical application to FLASH and European XFEL project

Wake potential of FLASH tapered collimator (set 1) made from copper (κ=58e6 S/m) for Gaussian bunch of σ = 50 μm .



	Set 1	Set 2	Set 3
b	2 mm	3 mm	6 mm
d	4.5 mm	5.5 mm	8 mm

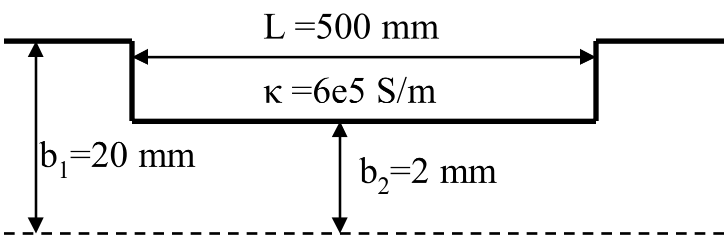




	Geometric				Full			
	Loss factor V/pC	Rms Loss factor V/pC	Kick factor V/pC/m	Rms Kick factor V/pC/m	Loss factor V/pC	Rms Loss factor V/pC	Kick factor V/pC/m	Rms Kick factor V/pC/m
Set 1	279.4	129.5	2037	900	292.4	141.2	2424	1114
Set 2	244.1	103.4	1123	594	253.3	113.4	1240	670
Set 3	157.5	63.6	266	157.9	163.4	69.5	280.7	167.3

Practical application to FLASH and European XFEL project

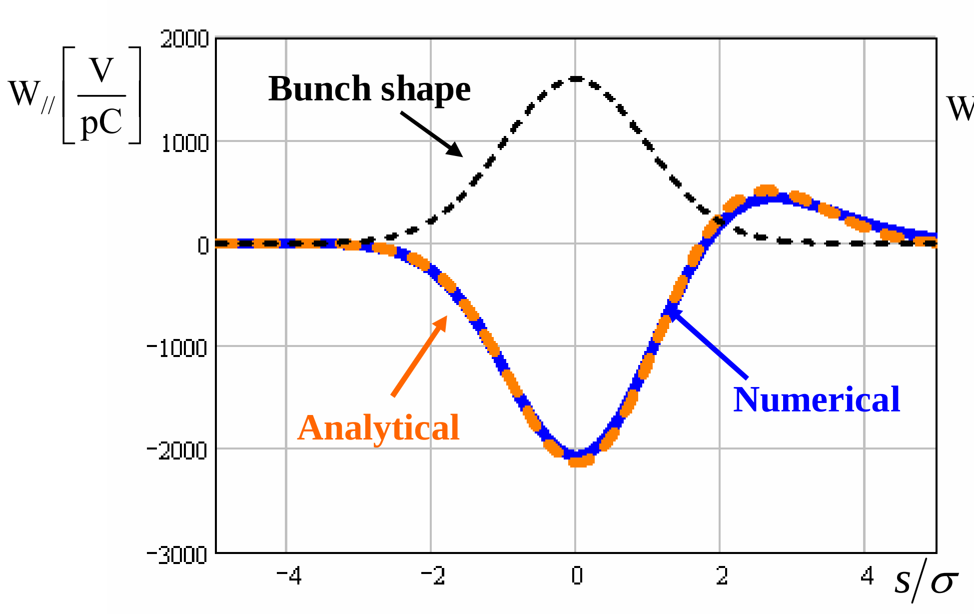
Wake potential of XFEL step collimator made from titanium (κ=0.6e6 S/m)
for Gaussian bunch of σ = 25μm .

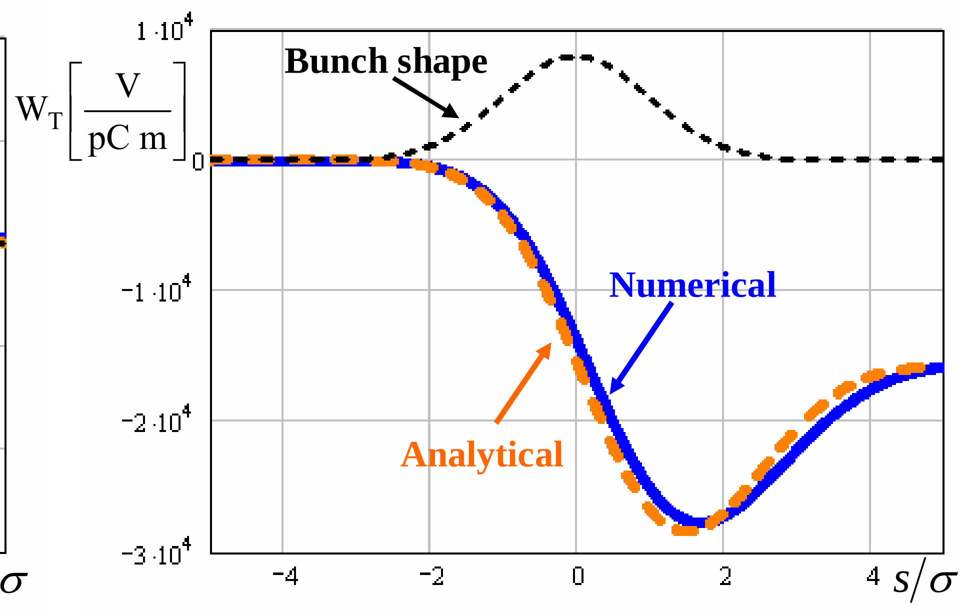


Geometric wakes

$$W_{//}(s) = \frac{Z_0 c}{\pi} \ln\left(\frac{b_1}{b_2}\right) \rho(s)$$

$$W_T(s) = \frac{Z_0 c}{\pi} \left(\frac{1}{b_2^2} - \frac{1}{b_1^2}\right) \theta(s)$$

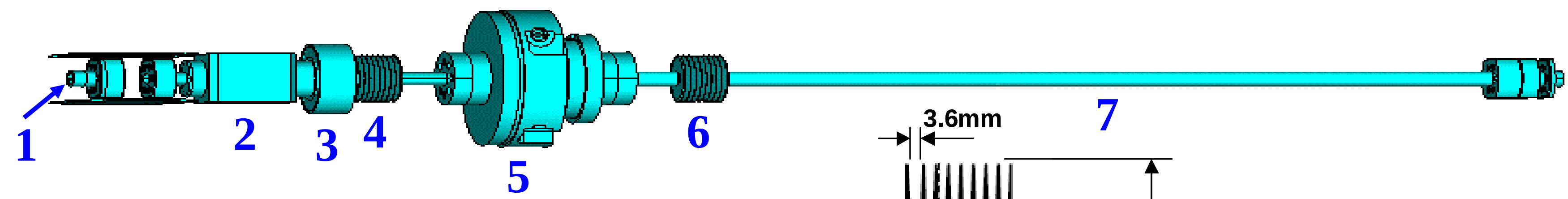




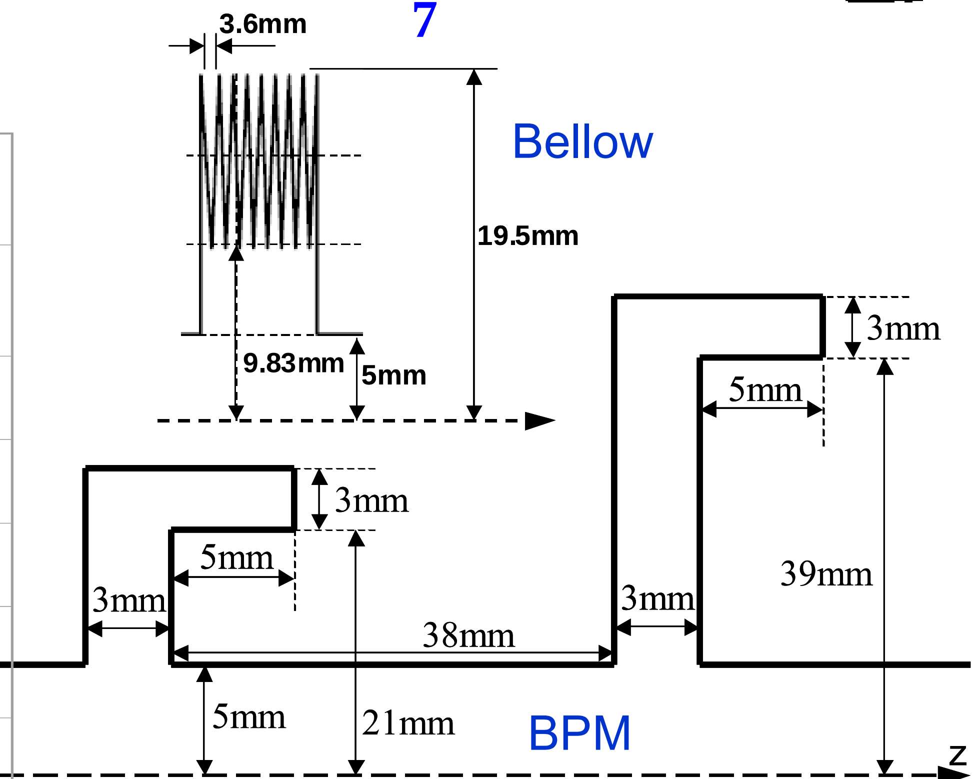
Loss factor kV/pC	Rms Loss factor kV/pC	Kick factor kV/pC/m	Rms Kick factor kV/pC/m
1.4	0.65	14	8.7

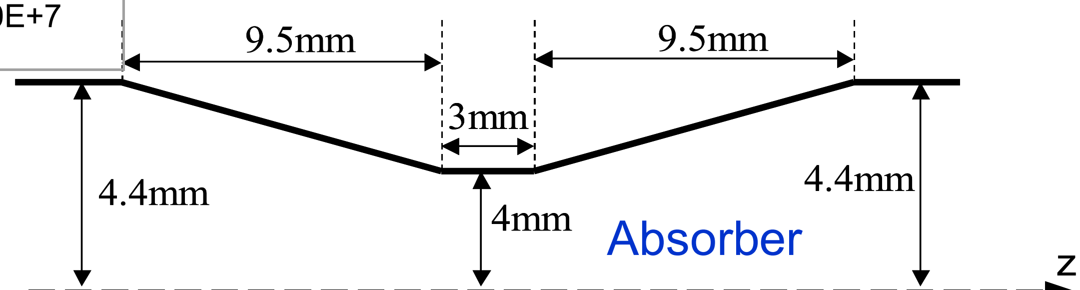
Practical application to FLASH and European XFEL project

Geometries of XFEL undulator intersection elements



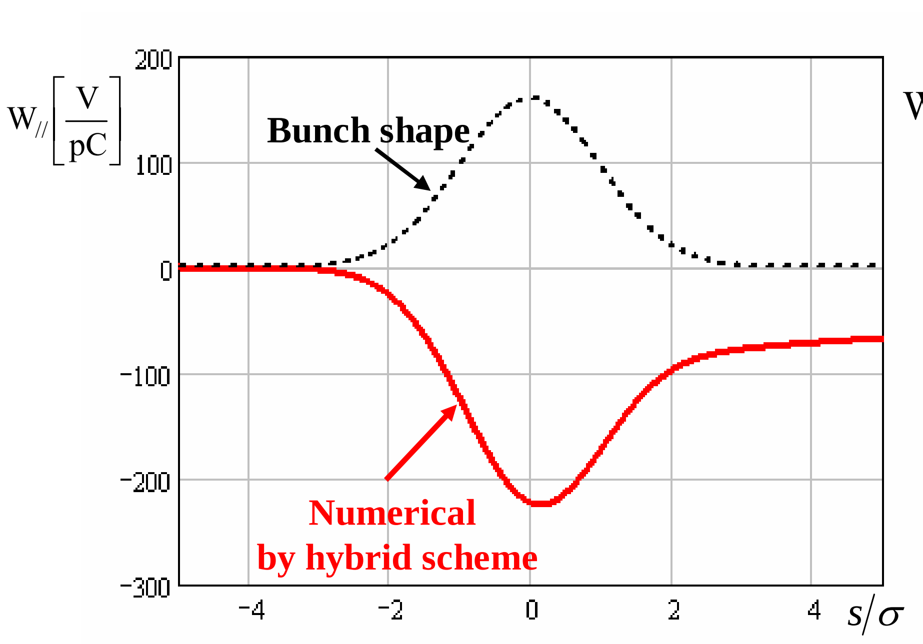
N	Element	Lenght	Material	Conducti vity
1	Elliptical pipe	5161 mm	Aluminum	3.66E+07
2	Pump	105 mm	Aluminum	3.66E+07
3	Absorber	22 mm	Copper	5.80E+7
4	Bellow	30 mm	BeCu 174	2.78E+7
5	BPM	51 mm	Stainless Steel 304	1.4E+6
6	Bellow	30 mm	BeCu 174	2.78E+7
7	Round pipe	652 mm	Copper	5.80E+7

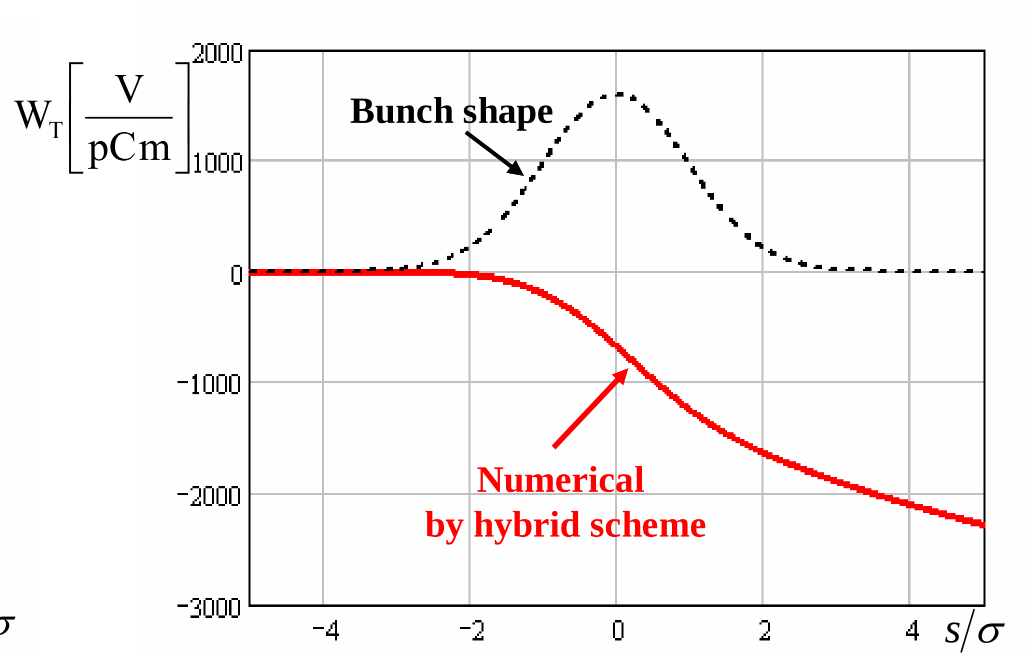




Practical application to FLASH and European XFEL project

Wake potentials calculated by hybrid numerical scheme





Loss factor V/pC	Rms Loss factor V/pC	Kick factor V/pC/m	Rms Kick factor V/pC/m
167.4	56.04	713.6	460.8

Summary

- New hybrid numerical scheme was developed to calculate wake fields excited by ultra short bunches in structures with finite resistivity. This scheme has no dispersion in longitudinal direction.

- Numerical tests and stability analyses of the new scheme was performed.The results obtained by hybrid scheme are in good agreement with analytical solutions and with result obtained by CST-PS.

- The hybrid scheme was used to calculate the wake potentials of FLASH tapered collimator. The results shows that the lass factor is increased by 4% and kick factor by 5-20% (set 1,2,3).

- It was shown that the wake potentials of the XFEL step collimator can be obtained as sum of geometric and resistive steady state wakes. In this case less than 3% of the transition wake will be neglected. Finally the hybrid scheme was used to calculate the wake potentials of XFEL undulator intersection. Obtained wakes includes the resistive transitions that was neglected in XFEL impedance budget so far.

Longitudinal dispersion free TE/TM numerical scheme

Field update in vacuum

$$\widehat{\mathbf{e}}_r^{n+0.5} = \widehat{\mathbf{e}}_r^{n-0.5} - \Delta\tau\, \mathbf{M}_{\varepsilon_r^{-1}} \mathbf{P}_z^* \widehat{\mathbf{h}}_\varphi^n$$

$$\widehat{\mathbf{h}}_\varphi^{n+0.5} = \widehat{\mathbf{h}}_\varphi^n + \frac{\Delta\tau}{2} \mathbf{M}_{\mu^{-1}} \left[\mathbf{P}_z \widehat{e}_r^{n+0.5} - \mathbf{P}_r \widehat{e}_z^n + \widehat{\mathbf{e}}_c^n \right]$$

$$\mathbf{W} \frac{\widehat{\mathbf{e}}_z^{n+1} - \widehat{\mathbf{e}}_z^n}{\Delta\tau} = \mathbf{M}_{\varepsilon_z^{-1}} \left[\mathbf{P}_r^* \widehat{\mathbf{h}}_\varphi^{n+0.5} + \widehat{\mathbf{j}}_z^{n+0.5} \right]$$

$$\widehat{\mathbf{h}}_\varphi^{n+1} = \widehat{\mathbf{h}}_\varphi^{n+0.5} + \frac{\Delta\tau}{2} M_{\mu^{-1}} \left[\mathbf{P}_z \widehat{e}_r^{n+0.5} - \mathbf{P}_r \widehat{e}_z^{n+1} + \widehat{\mathbf{e}}_c^{n+1} \right]$$

where $\mathbf{W} = \mathbf{I} + \frac{\Delta\tau^2}{4} \mathbf{M}_{\mu_\varphi^{-1}} \mathbf{P}_r \mathbf{M}_{\varepsilon_r^{-1}} \mathbf{P}_r^*$

Field update in conductor

$$\widehat{\mathbf{e}}_s^{\,n+1} = \mathbf{A}\mathbf{e}_s^{\,n} + \mathbf{B}\mathbf{P}_s \frac{\widehat{\mathbf{h}}_s^{\,n+1} + \widehat{\mathbf{h}}_s^{\,n}}{2}$$

$$\widehat{\mathbf{h}}_s^{\,n+1} = \widehat{\mathbf{h}}_s^{\,n} + \Delta\tau \mathbf{P}_s^* \frac{\widehat{\mathbf{e}}_s^{\,n+1} + \widehat{\mathbf{e}}_s^{\,n}}{2}$$

$$a_{ii} = e^{-\kappa Z_0 \Delta\tau}$$

$$a_{00} = \mathrm{e}^{-0.5\kappa Z_0 \Delta\tau}$$

$$b_{ii} = \frac{(1 - a_{ii})}{\kappa Z_0}$$

Algorithm Stability Condition

$$\Delta\tau \le \Delta z$$

Longitudinal Dispersion Free Condition

$$\Delta\tau = \Delta z$$

Algorithm Stability and Convergence

Leapfrog Algorithm | *TE/TM Splitting Algorithm*

Stability Condition for equidistance space grid

Leapfrog: $c\Delta t \le \frac{1}{\sqrt{3}} \Delta z$ — TE/TM Splitting: $c\Delta t \le \Delta z$

Longitudinal dispersion free condition

Leapfrog: $c\Delta t = \Delta z$ — TE/TM Splitting: $c\Delta t = \Delta z$

Contradiction ! (Leapfrog)

Compatible ! (TE/TM Splitting)

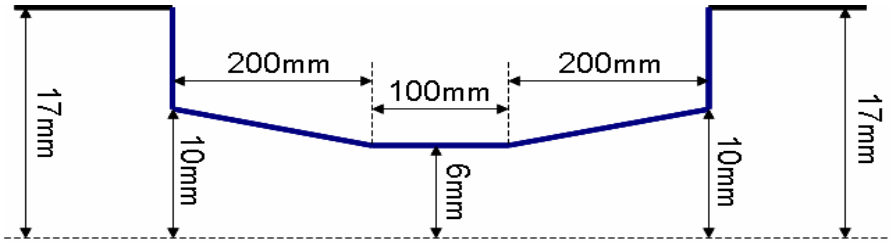
The Goal is to have ***Stable*** *and* ***Dispersion free*** *second order Algorithm*

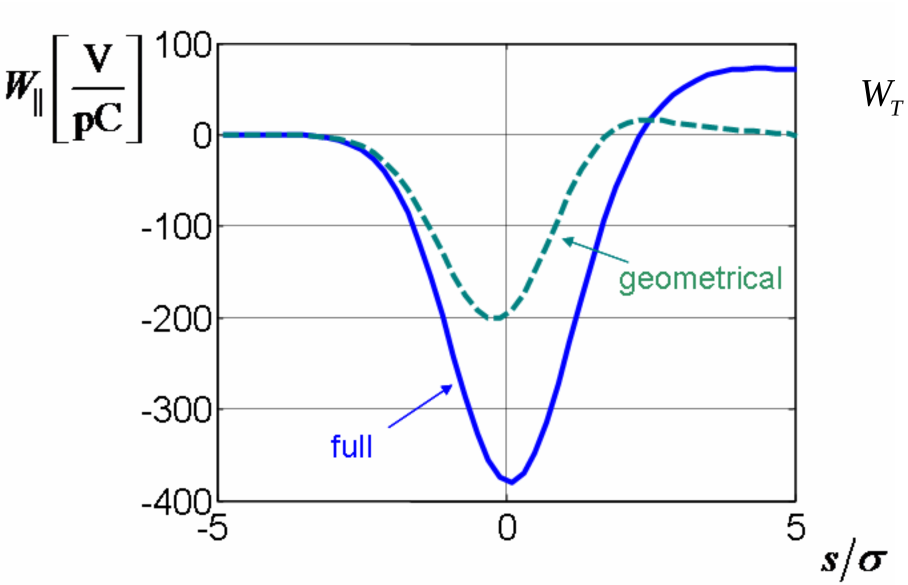
⇓

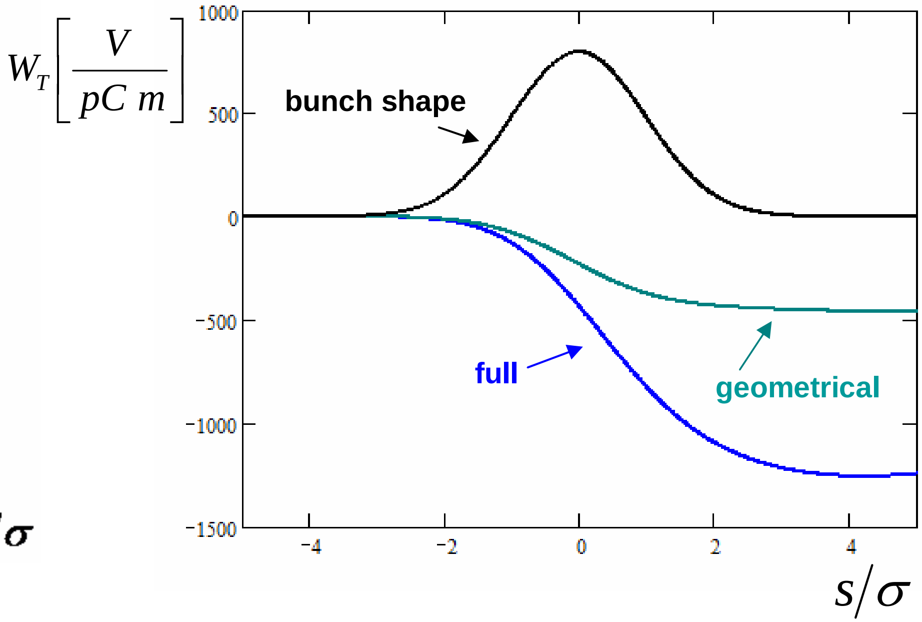
TE/TM Splitting Algorithm is applicable !!!

Numerical Examples

Comparison of wake potentials of tapered collimator "with" and "without" resistivity for Gaussian bunch σ = 50 μm .







Loss factor for finite conductive walls cannot be obtain as direct sum of the geometrical and the steady-state solution.